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FUNCTIONAL REPRESENTATIONS OF LATTICE-ORDERED SEMIRINGS. III

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Lattice-ordered semirings (*drl*-semirings) are considered. Compact sheaves of *drl*-semirings are defined and their characterization is obtained. The properties of compact sheaves are studied; in particular, the structure of irreducible and maximal l-ideals in the *drl*-semiring of sections of a compact sheaf is described. A compact sheaf of functional semirings (f-semirings) is described in terms of a continuous mapping of the irreducible (and maximal) spectrum of this sheaf onto a compact Hausdorff space. The paper also contains a proof that an f-semiring is Gelfand if and only if it is isomorphic to the semiring of all sections of a compact sheaf of f-semirings with a unique maximal ideal.

Keywords: lattice-ordered semiring, functional semiring, compact sheaf, Gelfand f-semiring.

MSC: 16Y60

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REFERENCES

1. Rao P.R. Lattice ordered semirings. *Math. Sem. Notes, Kobe Univ.*, 1981, vol. 9, pp. 119–149.
2. Swamy K.L.N. Dually residuated lattice ordered semigroups. *Math. Ann.*, 1965, vol. 159, pp. 105–114. doi: 10.1007/BF01360284 .
3. Chermnykh O.V. On *drl*-semigroups and *drl*-semirings. *Chebyshevskii Sb.*, 2016, vol. 17, no. 4, pp. 167–179 (in Russian). doi: 10.22405/2226-8383-2016-17-4-167-179 .
4. Chermnykh V.V., Chermnykh O.V. Functional representations of lattice-ordered semirings. *Siberian Electronic Mathematical Reports*, 2017, vol. 14, pp. 946–971 (in Russian). doi: 10.17377/semi.2017.14.080 .
5. Chermnykh O.V. Functional representations of lattice-ordered semirings. II. *Siberian Electronic Mathematical Reports*, 2018, vol. 15, pp. 677–684 (in Russian). doi: 10.17377/semi.2018.15.053 .
6. Pierce R.S. Modules over commutative regular rings. *Mem. Amer. Math. Soc.*, 1967, vol. 70, pp. 1–112. doi: 10.1090/memo/0070 .
7. Koh K. On a representation of strongly harmonic ring by sheaves. *Pacif. J. Math.*, 1972, vol. 41, no. 2, pp. 459–468. doi: 10.2140/pjm.1972.41.459 .
8. Mulvey C.J. A generalization of Gelfand duality. *J. Algebra*, 1979, vol. 56, no. 2, pp. 499–505. doi: 10.1016/0021-8693(79)90352-1 .
9. Filipoiu A. Compact sheaves of lattices and normal lattices. *Math. Jap.*, 1991, vol. 36, no. 2, pp. 381–386.
10. Chermnykh V.V. Representation of positive semirings by sections. *Russian Math. Surveys*, 1992, vol. 47, no. 5, pp. 180–182. doi: 10.1070/RM1992v047n05ABEH000948 .
11. Chermnykh V.V. Functional representations of semirings. *J. Math. Sci.*, 2012, vol. 187, no 2, pp. 187–267. doi: 10.1007/s10958-012-1062-2 .
12. Mulvey C.J. Compact ringed spaces. *J. Algebra*, 1978, vol. 52, no. 2, pp. 411–436. doi: 10.1016/0021-8693(78)90248-X .
13. Simmons H. Compact representations — the lattice theory of compact ringed spaces. *J. Algebra*, 1989, vol. 126, no. 2, pp. 493–531. doi: 10.1016/0021-8693(89)90315-3 .
14. Dauns J., Hofmann K.H. The representation of biregular rings by sheaves. *Math. Z.*, 1966, vol. 91, pp. 103–123. doi: 10.1007/BF01110158 .
15. Davey B.A. Sheaf spaces and sheaves of universal algebras. *Math. Z.*, 1973, vol. 134, no. 4, pp. 275–290. doi: 10.1007/BF01214692 .
16. Birkhoff G. *Lattice Theory*. Providence, R.I.: Amer. Math. Soc., 1967, 418 p. ISBN: 0-8218-1025-1 .

17. Miklin A.V., Chermnykh V.V., On *drl*-semirings. *Matematicheskiy vestnik pedvuzov i universitetov Volgo–Vyatskogo regiona*, 2014, vol. 16, pp. 87–95 (in Russian).

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