

ON THE PROBLEM OF SEQUENTIAL TRAVERSAL OF MEGALOPOLISES WITH PRECEDENCE CONDITIONS AND COST FUNCTIONS DEPENDING ON A LIST OF TASKS

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A constrained routing problem with complicated cost functions is studied. The construction of the cost functions can be difficult, and therefore the stages of this construction are elements of the solution of the problem. This situation arises, in particular, in studying the engineering problem of dismantling radiation hazardous elements, where, in the framework of a problem statement traditional for discrete optimization, it takes an unacceptably long time to construct a cost matrix whose entries characterize the radiation doses received by performers at the stage of displacement and dismantling. It is assumed that, at the stage of the computational implementation of the resulting optimal algorithm, the corresponding “parts” of the matrix may be not fed to the computer’s memory but calculated as needed. Possible applications of the developed methods may be related to the problem of dismantling a decommissioned generator unit of an NPP.

Keywords: dynamic programming, route, Bellman function.

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