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CRITERION OF SUBNORMALITY IN A FINITE GROUP: REDUCTION TO ELEMENTARY BINARY PARTITIONS

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Wielandt's criterion for the subnormality of a subgroup in a finite group is developed. For a set $\pi = \{p_1, p_2, \ldots, p_n\}$ and a partition $\sigma = \{\{p_1\}, \{p_2\}, \ldots, \{p_n\}, \{\pi\}'\}$, it is proved that a subgroup H is σ-subnormal in a finite group G if and only if it is $\{\{p_i\}, \{p_i\}'\}$-subnormal in G for every $i = 1, 2, \ldots, n$. In particular, H is subnormal in G if and only if it is $\{\{p\}, \{p\}'\}$-subnormal in $\langle H, H^x\rangle$ for every prime p and any element $x \in G$.

Keywords: finite group, subnormal subgroup, σ-subnormal subgroup, elementary binary partition.

MSC: 20D25, 20D35

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