TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 26 No. 3 **2020**

AUTOMORPHISMS OF THE SEMIRING OF POLYNOMIALS $\mathbb{R}_+^\vee[x]$ AND LATTICES OF ITS SUBALGEBRAS

V. V. Sidorov

A commutative semiring with zero and unity different from a ring where each nonzero element is invertible is called a semifield with zero. Let $\mathbb{R}_+^\vee$ be the semifield with zero of nonnegative real numbers with operations of max-addition and multiplication. For any positive real numbers a and s, denote by $\psi_{a,s}$ the automorphism of the semiring of polynomials $\mathbb{R}_+^\vee[x]$ defined by the rule $\psi_{a,s}\colon a_0 \vee a_1 x \vee \ldots \vee a_n x^n \mapsto a_0^s \vee a_1^s (ax) \vee \ldots \vee a_n^s (ax)^n$. It is proved that the automorphisms of the semiring $\mathbb{R}_+^\vee[x]$ are exactly the automorphisms $\psi_{a,s}$. The ring $C(X)$ of continuous $\mathbb{R}$-valued functions defined on an arbitrary topological space X is an algebra over the field $\mathbb{R}$ of real numbers. A subalgebra of $C(X)$ is any nonempty subset closed under addition and multiplication of functions and under multiplication by constants from $\mathbb{R}$. Similarly, we call a nonempty subset $A \subseteq \mathbb{R}_+^\vee[x]$ a subalgebra of $\mathbb{R}_+^\vee[x]$ if $f \vee g, fg, rf \in A$ for any $f, g \in A$ and $r \in \mathbb{R}_+^\vee$. It is proved that an arbitrary automorphism of the lattice of subalgebras of $\mathbb{R}_+^\vee[x]$ is induced by some automorphism of $\mathbb{R}_+^\vee[x]$. The same result also holds for the lattice of subalgebras with unity of the semiring $\mathbb{R}_+^\vee[x]$. The technique of one-generated subalgebras is applied.

Keywords: semiring of polynomials, lattice of subalgebras, automorphism, max-addition.

MSC: 06B05, 16S60, 54H99

DOI: 10.21538/0134-4889-2020-26-3-171-186

REFERENCES

1. Gelfand I.M., Kolmogorov A.N. On rings of continuous functions on topological spaces. *Dokl. Akad. Nauk SSSR*, 1939, vol. 22, no. 1, pp. 11–15.
2. Hewitt E. Rings of real-valued continuous functions. I. *Trans. Amer. Math. Soc.*, 1948, vol. 64, no. 1, pp. 45–99. doi: 10.1090/S0002-9947-1948-0026239-9 .
3. Vechtomov E.M. Lattice of subalgebras of the ring of continuous functions and Hewitt spaces. *Math. Notes*, 1997, vol. 62, pp. 575–580. doi: 10.1007/BF02361295 .
4. Sidorov V.V. Determinability of semirings of continuous nonnegative functions with max-plus by the lattices of their subalgebras. *Lobachevskii J. Math.*, 2019, vol. 40, pp. 90–100. doi: 10.1134/S1995080219010128 .
5. Sidorov V.V. Automorphisms of the lattice of all subalgebras of the semiring of polynomials in one variable. *J. Math. Sci.*, 2012, vol. 187, no. 2, pp. 169–176. doi: 10.1007/s10958-012-1060-4 .

Received May 2, 2020
Revised May 20, 2020
Accepted June 1, 2020

Vadim Veniaminovich Sidorov, Cand. Sci. (Phys.-Math.), Vyatka State University, Kirov, 610000 Russia, e-mail: sedoy_vadim@mail.ru .

V. V. Sidorov. Automorphisms of the semiring of polynomials $\mathbb{R}_+^\vee[x]$ and lattices of its subalgebras, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2020, vol. 26, no. 3, pp. 171–186 .