

ON ITERATIVE METHODS OF FINDING THE EQUILIBRIUM IN THE ARROW–DEBREU CLASSICAL MODEL OF PURE EXCHANGE WITH MULTIPLICATIVE UTILITY FUNCTIONS OF THE PARTICIPANTS

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For the classical Arrow–Debreu exchange models with multiplicative utility functions of the participants, new iterative schemes for setting the equilibrium prices are proposed. Each iteration of the new algorithms corresponds to one exchange cycle. During each cycle, the participants respond to current market prices and exchange goods based on their budgets and their preference systems. The only observations available to the participants are the disappearance from the market of certain products that pass into the category of scarce ones. This forces the exchange participants to adjust the prices for such goods. Namely, the prices corresponding to the goods that have become scarce grow by some relatively constant value. At the same time, other prices, including the prices of commodities remaining in excess, do not change. Because of this, the total level of prices gradually increases (which corresponds to the normal inflation observed in any market economy). The growth of prices forces a reduction in the excessive demand for scarce goods and its switching to other product groups, in accordance with the existing norms of their interchangeability. Although the growth of prices is fixed, their overall growth from iteration to iteration leads to the fact that not absolute but relative changes gradually fade, providing a generalized convergence of the iterative process. As a convergent sequence, it is possible to track the so-called normalized prices. The corresponding convergence theorems and results of numerical experiments are presented, including cases of other types of economies, up to the most extravagant.

Keywords: economic equilibrium, exchange model, multiplicative utility function, coordinate descent methods.

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