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1-LATTICE ISOMORPHISMS OF MONOIDS DECOMPOSABLE INTO A FREE PRODUCT

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Let M and M' be monoids. Denote by $\mathrm{Sub}^1 M$ the lattice of all submonoids of M. Any isomorphism of $\mathrm{Sub}^1 M$ onto the lattice $\mathrm{Sub}^1 M'$ is called a 1-*lattice isomorphism* of M onto M'. We say that a bijection φ of M onto M' *induces* a 1-lattice isomorphism ψ of M onto M' if $\varphi(K) = \psi(K)$ for any submonoid $K \in \mathrm{Sub}^1 M$. A monoid M *is strictly* 1-*lattice determined* if any of its 1-lattice isomorphisms onto an arbitrary monoid is induced either by an isomorphism or by an antiisomorphism. The similar notions of a group strictly determined by its subgroup lattice and a semigroup strictly determined by its subsemigroup lattice have long attracted attention and have been actively studied in the classes of groups and semigroups. For monoids almost nothing has been known here. However, the following question was asked about forty years ago: is any monoid that is decomposable into a free product strictly 1-lattice determined? A complete answer to this question is found. Namely, it is proved that an arbitrary monoid nontrivially decomposable into a free product is strictly 1-lattice determined. This result has something in common with the well-known results on the strictly lattice determinability of both a group nontrivially decomposable into a free product and a semigroup decomposable into a free product.

Keywords: monoid, submonoid lattice, free product, 1-lattice isomorphism.

MSC: 20M32

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