TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 26 No. 3 **2020**

INVERSE PROBLEMS IN THE CLASS OF Q-POLYNOMIAL GRAPHS

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In the class of distance-regular graphs Γ of diameter 3 with a pseudogeometric graph Γ_3, feasible intersection arrays for the partial geometry were found for networks by Makhnev, Golubyatnikov, and Guo; for dual networks by Belousov and Makhnev; and for generalized quadrangles by Makhnev and Nirova. These authors obtained four infinite series of feasible intersection arrays of distance-regular graphs:

$$\{c_2(u^2-m^2)+2c_2m-c_2-1, c_2(u^2-m^2),\ (c_2-1)(u^2-m^2)+2c_2m-c_2; 1, c_2, u^2-m^2\},$$

$$\{mt, (t+1)(m-1), t+1; 1, 1, (m-1)t\} \quad \text{for} \quad m \le t,$$

$$\{lt, (t-1)(l-1), t+1; 1, t-1, (l-1)t\}, \quad \text{and} \quad \{a(p+1), ap, a+1; 1, a, ap\}.$$

We find all feasible intersection arrays of Q-polynomial graphs from these series. In particular, we show that, among these infinite families of feasible arrays, only two arrays ($\{7, 6, 5; 1, 2, 3\}$ (folded 7-cube) and $\{191, 156, 153; 1, 4, 39\}$) correspond to Q-polynomial graphs.

Keywords: distance-regular graph, Q-polynomial graph, graph Γ with a strongly regular graph Γ_3.

MSC: 05C25

DOI: 10.21538/0134-4889-2020-26-3-14-22

REFERENCES

1. Bang S., Koolen J. Distance-regular graphs of diameter three having eigenvalue -1. *Linear Algebra and its Applications*, 2017, vol. 531, pp. 38–53. doi: 10.1016/j.laa.2017.05.038 .
2. Iqbal Q., Koolen J., Park J., Rehman M. Distance-regular graphs with diameter 3 and eigenvalue $a_2 - c_3$. *Linear Algebra and Appl.*, 2020, vol. 587, pp. 271–290. doi: 10.1016/j.laa.2019.10.021 .
3. Makhnev A.A., Golubyatnikov M.P., Guo Wenbin. Inverse problems in distance-regular graphs: nets. *Communications in Mathematics and Statistics*, 2019, vol. 7, no. 1, pp. 69–83. doi: 10.1007/S40304-018-0159-4 .
4. Makhnev A.A., Nirova M.S. Inverse problems in distance-regular graphs: generalized quadrangles. *Sibirean Electr. Math. Reports*, 2018, vol. 15, pp. 927–934. doi: 10.17377/semi.2018.15.079 .
5. Brouwer A.E., Cohen A.M., Neumaier A. *Distance-Regular Graphs*. Berlin; Heidelberg; N Y: Springer-Verlag, 1989, 495 p. ISBN: 0387506195 .
6. Terwilliger P. A new inequality for distance-regular graphs. *Discrete Mathematics*, 1995, vol. 137, pp. 319–332. doi: 10.1016/0012-365X(93)E0170-9 .
7. Jurisic A., Vidali J. Extremal 1-codes in distance-regular graphs of diameter 3. *Des. Codes Cryptogr.*, 2012, vol. 65, no. 1-2, pp. 29–47. doi: 10.1007/s10623-012-9651-0 .

Received May 22, 2020
Revised June 17, 2020
Accepted July 13, 2020

Funding Agency: This work was supported by the Russian Foundation for Basic Research – the National Natural Science Foundation of China (project no. 20-51-53013_a).

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I. N. Belousov, A. A. Makhnev. Inverse problems in the class of Q-polynomial graphs, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2020, vol. 26, no. 3, pp. 14–22 .