TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN
**Vol. 26 No. 3** **2020**

# TENSOR REPRESENTATIONS AND GENERATING SETS OF INVOLUTIONS OF SOME MATRIX GROUPS

**Ya. N. Nuzhin**

It is well known that all irreducible representations of Chevalley groups over infinite fields and modular representations in nice characteristics of fields of definition are exhausted by subrepresentations of tensor products of their natural representations. We consider two specific subrepresentations of this kind and use them to answer two questions on the number of generating involutions of some matrix groups. For an integral domain $D$ of characteristic different from 2, we establish the irreducibility of the symmetric and external squares of the natural representation of the group $SL_n(D)$ and find their kernels (Theorem 1). Denote by $n(G)$ (by $n_c(G)$) the minimum number of generating (and also conjugate, respectively) involutions of $G$ whose product is 1. Problems on finding the numbers $n(G)$ and $n_c(G)$ for finite simple groups are written by the author in the *Kourovka Notebook* (Question 14.69). Based on Theorem 1 and L. L. Scott's inequality, we prove the following result. Let $G$ be $SL_3(D)$ or $SL_6(D)$, where $D$ is an integral domain of characteristic different from 2. Then $n(G) > 5$ and, in particular, $G$ is not generated by three involutions two of which commute; moreover, if $D$ is the ring of integers or a finite field (of odd order), then $n(G) = n_c(G) = 6$ (Theorem 2).

Keywords: special linear group over the integral domain, tensor representations, generating sets of involutions.

**MSC:** 20G05, 20G15

**DOI**: 10.21538/0134-4889-2020-26-3-133-141

## REFERENCES

1. Steinberg R. *Lectures on Chevalley groups.* Providence: AMS, 2016, 160 p. ISBN: 9781470431051. Translated to Russian under the title *Lektsii o gruppakh Shevalle*, Moscow: Mir Publ., 1975. 263 p.
2. *The Kourovka notebook. Unsolved problems in group theory*, 17th ed., eds. V.D. Mazurov and E.I. Khukhro, Inst. Math. SO RAN, Novosibirsk, 2010, 136 p. ISBN: 9785943568916.
3. Nuzhin Ya.N. Generating sets of involutions of finite simple groups. *Algebra Logika*, 2019, vol. 58, no. 3, pp. 426–434 (in Russian). doi: 10.33048/alglog.2019.58.310.
4. Scott L.L. Matricies and cohomology. *Ann. of Math.*, 1977, vol. 105, no. 3, pp. 473–492.
5. Tamburini M.C., Zucca P. Generation of certain matrix groups by three involutions, two of which commute. *J. Algebra*, 1997, vol. 195, no. 2, pp. 650–661. doi: 10.1006/jabr.1997.7055.
6. Nuzhin Ya.N. Generating triples of involutions of groups of Lie type over a finite field of odd characteristic. II. *Algebra Logic*, 1997, vol. 36, no. 4, pp. 245–256. doi: 10.1007/s10469-997-0066-3.
7. Nuzhin Ya.N. On the generability of the group $PSL_n(Z)$ by three involutions, two of which commute. *Vladikavkaz. Mat. Zh.*, 2008, vol. 10, no. 1, pp. 68–74 (in Russian).
8. Levchuk D.V., Nuzhin Ya.N. On generation of the group $PSL_n(\mathbb{Z} + i\mathbb{Z})$ by three involutions, two of which commute. *J. Sib. Fed. Univ. Math. Phys.*, 2008, vol. 1, no. 2, pp. 133–139 (in Russian).
9. Ward J.M. *Generation of simple groups by conjugate involutions.* Queen Mary college, University of London, Thesis of Doctor of Philosophy, 2009, 193 p.
10. Diedonne J. Les determinants sur up corps non commutatif. *Bull. Soc. Math. France*, 1943, vol. 71, pp. 27–45. doi: 10.24033/bsmf.1345.
11. Suprunenko D.A. *Matrix groups.* Providence: AMS, 1976, 252 p. ISBN: 0821813412. Original Russian text published in Suprunenko D.A. *Gruppy matrits.* Moscow: Nauka Publ., 1972, 352 p.
12. Moiseenkova T.V. Generating multiplets of involution of the groups $SL_n(\mathbb{Z})$ and $PSL_n(\mathbb{Z})$. *Trudy Inst. Mat. i Mekh. UrO RAN*, 2010, vol. 16, no. 3, pp. 195–198 (in Russian).
13. Gillio B.M., Tamburini M.C. Some class of groups generated by three involutions. *Istit. Lombardo Accad. Sci. Lett. Rend. A*, 1985, vol. 116, pp. 191–209 (in Italian).

14. Nuzhin Ya.N. Generating triples of involutions of Chevalley groups over a finite field of characteristic 2. *Algebra and Logic*, 1990, vol. 29, no. 2, pp. 134–143. doi: 10.1007/BF02001358 .
15. Vsemirnov M.A. Is the group SL(6, $\mathbb{Z}$) $(2,3)$-generated? *J. Math. Sci. (N.Y.)*, 2007, vol. 140, no. 5, pp. 660–675. doi: 10.1007/s10958-007-0006-8 .
16. Vsemirnov M.A. On (2,3)-generation of matrix groups over the ring of integers. *St. Petersburg Math. J.*, 2008, vol. 19, no. 6, pp. 883–910. doi: 10.1090/S1061-0022-08-01026-1 .
17. Di Martino L., Vavilov N. $(2,3)$-generation of $SL(n,q)$. I. Cases $n = 5, 6, 7$. *Comm. Algebra*, 1994, vol. 22, no. 4, pp. 1321–1347. doi: 10.1080/00927879408824908 .
18. Tabakov K., Tchakerian K. $(2,3)$-generation of the groups $PSL_6(q)$. *Serdica Math. J.*, 2011, vol. 37, no. 4, pp. 365–370.
19. Pellegrini M.A. The $(2,3)$-generation of the special linear groups over finite fields. *Bull. Aust. Math. Soc.*, 2017, vol. 95, no. 1, pp. 48–53. doi: 10.1017/s0004972716000617 .



Received May 10, 2020
Revised July 6, 2020
Accepted July 20, 2020

**Funding Agency:** This work was supported by the Krasnoyarsk Mathematical Center, which is financed by the Ministry of Science and Higher Education of the Russian Federation within the project for the establishment and development of regional centers for mathematical research and education (agreement no. 075-02-2020-1534/1), and by the Russian Foundation for Basic Research (project no. 19-01-00566).



*Yakov Nifant'evich Nuzhin*, Dr. Phys.-Math. Sci., Prof., Siberian Federal University, pr. Svobodnyi 79, Krasnoyarsk, 660041 Russia, e-mail: nuzhin2008@rambler.ru .



Cite this article as: Ya. N. Nuzhin. Tensor representations and generating sets of involutions of some matrix groups, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2020, vol. 26, no. 3, pp. 133–141 .