TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 26 No. 3 **2020**

A CRITERION FOR THE EXISTENCE OF NONDESTRUCTIVE CONTROLS IN THE PROBLEM OF OPTIMAL EXPLOITATION OF AN ECOSYSTEM WITH A BINARY STRUCTURE

Vl. D. Mazurov, A. I. Smirnov

Earlier the authors proved the equivalence of a proposed formulation of a renewable ecoresource sustainable exploitation problem (based on representing the exploited ecosystem by a discrete dynamic system) and a certain mathematical program. In this paper we prove the concavity of a map describing the dependence of the state vector of the ecosystem on the control in the case where the step operator of the dynamic system is concave. In the particular case of a structured ecosystem described by Leslie's binary model, conditions for the objective function are characterized under which there are optimal controls preserving all structural divisions of the system. In this case, we used the notion of local irreducibility, which generalizes the classical notion of map irreducibility.

Keywords: rational exploitation of ecosystems, optimal nondestructive controls, irreducible map, concave programming.

MSC: 47N05, 37N25, 37N40

DOI: 10.21538/0134-4889-2020-26-3-101-117

REFERENCES

1. *The state of world fisheries and aquaculture 2018. Meeting the sustainable development goals.* FAO-2018. Rome: UNO, 2018, 210 p. ISBN: 978-92-5-130562-1 .
2. *The state of the world's forests 2018. Meeting the sustainable development goals.* FAO-2018. Rome: UNO, 2018, 118 p. ISBN: 978-92-5-130561-4 .
3. Wallmo K., Bisack K.D., Lew D.K., Squires D.E. Editorial: The Economics of protected marine species: Concepts in research and management. *Front. Mar. Sci.*, 2016, vol. 3, art.-no. 183, 2 p. doi: 10.3389/fmars.2016.00183 .
4. De Lara M., Doyen L. *Sustainable management of natural resources: Mathematical models and methods.* Berlin; Heidelberg: Springer-Verlag, 2008, 266 p. doi: 10.1007/978-3-540-79074-7 .
5. Usher M.B. A matrix approach to the management of renewable resources, with special reference to selection forests. *J. Appl. Ecol.*, 1966, vol. 3, no. 2, pp. 355–367. doi: 10.2307/2401258 .
6. Williamson M.H. Introducing students to the concepts of population dynamics. In: J.M. Lambert (ed.) *Proc. of the British Ecological Society symposium The Teaching of Ecology.* Oxford: Blackwell, 1967, pp. 169–175. ISBN: 978-0632025701 .
7. Dunkel G.M. Maximum sustainable yields. *SIAM J. Appl. Math.*, 1970, vol. 19, no. 2, pp. 367–378. doi: 10.1137/0119035 .
8. Doubleday W.G. Harvesting in matrix population model. *Biometrics*, 1975, vol. 31, no. 1, pp. 189–200. doi: 10.2307/2529719 .
9. Reed W.J. Optimum age-specific harvesting in a nonlinear population model. *Biometrics*, 1980, vol. 36, no. 4, pp. 579–593. doi: 10.2307/2556112 .
10. Getz W.M. The ultimate-sustainable-yield problem in nonlinear age-structured populations. *Math. Biosci.*, 1980, vol. 48, no. 3-4, pp. 279–292. doi: 10.1016/0025-5564(80)90062-0 .
11. Grey D.R. Harvesting under density-dependent mortality and fecundity. *J. Math. Biol.*, 1988, vol. 26, no. 2, pp. 193–197. doi: 10.1007/BF00277732 .
12. Getz W.M., Haight R.G. *Population harvesting: Demographic models of fish,forest, and animal resources.* Princeton, New Jersey: Princeton University Press, 1989, 391 p. ISBN: 9780691085166 .

13. Caswell H. *Matrix population models: Construction, analysis, and interpretation.* 2nd ed. Sunderland, Massachusetts: Sinauer Associates, Inc. Publishers, 2001, 722 p. ISBN: 0-87893-096-5 .
14. Lefkovitch L.P. The study of population growth in organisms grouped by stages. *Biometrics*, 1965, vol. 21, no. 1, pp. 1–18. doi: 10.2307/2528348 .
15. Caswell H., de Vries C., Hartemink N., Roth G., van Daalen S.F. Age-stage classified demographic analysis: a comprehensive approach. *Ecological Monographs*, 2018, vol. 88, no. 4, pp. 560–584. doi: 10.1002/ecm.1306 .
16. Goodman L.A. The analysis of population growth when the birth and death rates depend upon several factors. *Biometrics*, 1969, vol. 25, no. 4, pp. 659–681. doi: 10.2307/2528566 .
17. Rogers A. *Applied Multiregional Demography: Migration and Population Redistribution.* Cham: Springer, 2015, 114 p. doi: 10.1007/978-3-319-22318-6 .
18. Frisman E.Ya., Kulakov M.P., Revutskaya O.L., Zhdanova O.L., Neverova G.P. The key approaches and review of current researches on dynamics of structured and interacting populations. *Computer Research and Modeling*, 2019, vol. 11, no. 1, pp. 119–151 (in Russian). doi: 10.20537/2076-7633-2019-11-1-119-151 .
19. Smirnov A.I. On some nonlinear generalization of the Leslie model considering the effect of saturation. *Vestnik Ural'skogo Instituta Ekonomiki, Upravleniya i Prava*, 2010, no. 4 (13), pp. 98–101 (in Russian).
20. Mazurov V.D., Smirnov A.I. On the reduction of the optimal non-destructive system exploitation problem to the mathematical programming problem. In: Yu.G. Evtushenko, M.Yu. Khachay, O.V. Khamisov, Yu.A. Kochetov, V.U. Malkova, M.A. Posypkin (eds). *Proc. of VIII Internat. Conf. on Optimization and Applications* (OPTIMA-2017), 2017, pp. 392–398.
21. Mazurov V.D., Smirnov A.I. Properties of admissible set of an optimal non-destructive system exploitation problem in some general formalization. In: S. Belim, A. Kononov, Yu. Kovalenko (eds). *Proc. of the School-Seminar on Optimization Problems and their Applications* (OPTA-SCL 2018). Omsk, Russia, 2018, pp. 359–371.
22. Smirnov A.I., Mazurov V.D. On existence of optimal non-destructive controls for ecosystem exploitation problem applied to a generalization of Leslie model. In: Yu.G. Evtushenko, M. Jaćimović, M.Yu. Khachay et al. (eds). *Proc. of IX Internat. Conf. on Optimization and Applications* (OPTIMA-2018) (Supplementary Volume), DEStech Transactions on Computer Science and Engineering, 2018, pp. 199–213. doi: 10.12783/dtcse/optim2018/27933 .
23. Rockafellar R. *Convex Analysis*. Princeton: Princeton University Press, 1970, 451 p. ISBN: 0691015864 . Translated to Russian under the title *Vypuklyi analiz*. Moscow: Mir Publ., 1973, 469 p.
24. Nikaido H. *Convex Structures and Economic Theory*. New York: Academic Press, 1968, 405 p. ISBN: 9781483230030 . Translated to Russian under the title *Vypuklye struktury i matematicheskaya ekonomika*. Moscow: Mir Publ., 1972, 518 p.
25. Mazurov V.D., Smirnov A.I. Generalization of Controls Bimodality Property in the Optimal Exploitation Problem for Ecological Population with Binary Structure. In: M. Jaćimović, M. Khachay, V. Malkova, M. Posypkin (eds), *Optimization and Applications*, 2020, Communications in Computer and Information Science, vol. 1145, Cham: Springer, pp. 206–221. doi: 10.1007/978-3-030-38603-0_16 .

Received June 11, 2020
Revised July 20, 2020
Accepted July 24, 2020

Funding Agency: This work was supported by the Russian Foundation for Basic Research (project no. 19-07-01243).

Vladimir Danilovich Mazurov, Dr. Phys.-Math. Sci. Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: mazurov@imm.uran.ru .

Aleksandr Ivanovich Smirnov, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: asmi@imm.uran.ru .

Vl. D. Mazurov, A. I. Smirnov. A criterion for the existence of nondestructive controls in the problem of optimal exploitation of an ecosystem with a binary structure, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2020, vol. 26, no. 3, pp. 101–117 .