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ON CONNECTED COMPONENTS OF FRACTAL CUBES

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The paper shows an essential difference between fractal squares and fractal cubes. The topological classification of fractal squares proposed in 2013 by K.-S. Lau et al. was based on analyzing the properties of the $\mathbb{Z}^2$-periodic extension $H = F + \mathbb{Z}^2$ of a fractal square F and of its complement $H^c = \mathbb{R}^2 \setminus H$. A fractal square $F \subset \mathbb{R}^2$ contains a connected component different from a line segment or a point if and only if the set H^c contains a bounded connected component. We show the existence of a fractal cube F in $\mathbb{R}^3$ for which the set $H^c = \mathbb{R}^3 \setminus H$ is connected whereas the set Q of connected components K_α of F possesses the following properties: Q is a totally disconnected self-similar subset of the hyperspace $C(\mathbb{R}^3)$, it is bi-Lipschitz isomorphic to the Cantor set $C_{1/5}$, all the sets $K_\alpha + \mathbb{Z}^3$ are connected and pairwise disjoint, and the Hausdorff dimensions $\dim_H(K_\alpha)$ of the components K_α assume all values from some closed interval $[a, b]$.

Keywords: fractal square, fractal cube, superfractal, self-similar set, hyperspace, Hausdorff dimension.

REFERENCES

1. Barnsley M.F., Hutchinson J.E., Stenflo Ö. V-variable fractals: Fractals with partial self similarity. *Adv. Math.*, 2008, vol. 218, no. 6, pp. 2051–2088. doi: 10.1016/j.aim.2008.04.011 .
2. Bonk M., Merenkov S. Quasisymmetric rigidity of square Sierpinski carpets. *Annals Math.*, 2013, vol. 177, no. 2, pp. 591–643. doi: 10.4007/annals.2013.177.2.5 .
3. Cristea L.L., Steinsky B. Curves of infinite length in 4 × 4-labyrinth fractals. *Geom. Dedicata*, 2009, vol. 141, pp. 1–17. doi: 10.1007/s10711-008-9340-3 .
4. Cristea L.L., Steinsky B. Curves of infinite length in labyrinth fractals. *Proc. Edinb. Math. Soc., II. Ser.*, 2011, vol. 54, no. 2, pp. 329–344. doi: 10.1017/S0013091509000169 .
5. Falconer K.J. *Fractal geometry: mathematical foundations and applications.* N Y: J. Wiley and Sons, 1990, 288 p. ISBN: 0471922870 .
6. Hutchinson J. Fractals and self-similarity. *Indiana Univ. Math. J.*, 1981, vol. 30, no. 5, pp. 713–747. DOI: 10.1512/iumj.1981.30.30055 .
7. Lau K.S., Luo J.J., Rao H. Topological structure of fractal squares. *Math. Proc. Camb. Phil. Soc.*, 2013, vol. 155, no. 1, pp. 73–86. doi: 10.1017/S0305004112000692 .
8. Luo J.J., Liu J.C. On the classification of fractal squares. *Fractals*, 2016, vol. 24, no. 1, art.-no. 1650008. doi: 10.1142/S0218348X16500080 .
9. Ruan H.J., Wang Y. Topological invariants and Lipschitz equivalence of fractal squares. *J. Math. Anal. Appl.*, 2017, vol. 451, no. 1, pp. 327–344. doi: 10.1016/j.jmaa.2017.02.012 .
10. Tetenov A.V. Finiteness properties for self-similar sets. *arXiv*:2003.04202 [math.MG].
11. Tetenov A.V., Drozdov D.A. On the connected components of fractal cubes. *arXiv*:2002.02920 [math.MG]. 6 p.

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