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ON THE PROPERTIES OF IRREDUCIBLE REPRESENTATIONS OF SPECIAL LINEAR AND SYMPLECTIC GROUPS THAT ARE NOT LARGE WITH RESPECT TO THE FIELD CHARACTERISTIC AND REGULAR UNIPOTENT ELEMENTS FROM SUBSYSTEM SUBGROUPS

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We study the properties of irreducible representations of special linear and symplectic groups that are not large with respect to the ground field characteristic and regular unipotent elements of nonprime order from subsystem subgroups of types A_l and C_l, respectively, with certain conditions on l. Assume that K is an algebraically closed field of characteristic $p > 2$, $G = A_r(K)$ or $C_r(K)$, $l < r - 1$ for $G = A_r(K)$ and $l < r$ for $G = C_r(K)$, $H \subset G$ is a subsystem subgroup with two simple components H_1 and H_2 of types A_l and A_{l-r-1} or C_l and C_{r-l}, respectively, and x is a regular unipotent element from H_1. Suppose that $l + 1 = ap^s + b$ for $G = A_r(K)$ and $2l = ap^s + b$ for $G = C_r(K)$ where $a < p$, $p \le b \le p^s$, and $s > 1$. An irreducible representation φ of G is said to be (p, x)-special if all the weights of the restriction of φ to a nice A_1-subgroup containing x^{p^s} are less than p (here the set of weights of a group of type A_1 is canonically identified with the set of integers). Denote by $d_\rho(z)$ the minimal polynomial of the image of an element z in a representation ρ and call the composition factor ψ of the restriction of φ to H large for $z \in H$ if $d_\psi(z) = d_\varphi(z)$. The main results of the paper are Theorems 1 and 2.

Theorem 1. Let φ be a (p, x)-special representation of G. Then the restriction of φ to H has no composition factors that are large for x and nontrivial for H_2.

Theorem 2. Under the assumptions of Theorem 1, the number of maximum size Jordan blocks of the element $\varphi(x)$ does not exceed a certain integer which depends only upon p, b, and the coefficients at the highest weight and does not depend on the group rank.

We explain why the case studied here should be considered separately. For instance, for p-restricted representations of the corresponding groups with large highest weights with respect to the characteristic, assertions opposite to Theorems 1 and 2 are valid. The results on the block structure of the images of unipotent elements in representations of algebraic groups can be used for solving recognition problems for representations and linear groups based of the presence of certain special matrices.

Keywords: unipotent elements, Jordan block sizes, special linear group, symplectic group.

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