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EXACT SOLUTION OF NAVIER–STOKES EQUATIONS DESCRIBING SPATIALLY INHOMOGENEOUS FLOWS OF A ROTATING FLUID

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We study an overdetermined system consisting of the Navier–Stokes equations and the incompressibility equation. The system of equations describes steady spatially inhomogeneous shear flows of a viscous incompressible fluid. The nontrivial exact solution of the system under consideration is determined in the Lin–Sidorov–Aristov class. A condition for the solvability of the system for the velocity field of the form

$$V_x = U(z) + u_1(z)\,x + u_2(z)\,y, \quad V_y = V(z) + v_1(z)\,x + v_2(z)\,y, \quad V_z = 0$$

is obtained. In the study of the exact solution, it is stated that the solvability of the system of equations is possible under an algebraic connection between the horizontal gradients (spatial accelerations) of the velocities u_1, u_2, v_1, v_2 and the pressure components P_{11}, P_{12}, P_{22}. Pressure is a quadratic form with respect to the coordinates x and y. It is established that the pressure components and spatial accelerations are constant. In this case, depending on the values of the parameters, an exact solution is obtained for the velocities U and V. The exact solutions obtained can describe the inhomogeneous Poiseuille–Couette–Ekman flow.

Keywords: layered flows, shear flows, exact solutions, Coriolis parameter, overdetermined system, compatibility conditions.

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