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BIPARTITE THRESHOLD GRAPHS

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A triple of distinct vertices (x, v, y) in a graph $G = (V, E)$ such that $xv \in E$ and $vy \notin E$ is called *lifting* if $\deg(x) \leq \deg(y)$ and *lowering* if $\deg(x) \geq 2 + \deg(y)$. A transformation φ of a graph G that replaces G with $\varphi(G) = G - xv + vy$ is called an *edge rotation* corresponding to a triple of vertices (x, v, y). For a lifting (lowering) triple (x, v, y), the corresponding edge rotation is called *lifting* (*lowering*). An edge rotation in a graph G is lifting if and only if its inverse in the graph $\varphi(G)$ is lowering. A bipartite graph $H = (V_1, E, V_2)$ is called a *bipartite threshold graph* if it has no lifting triples such that $x, y \in V_1$ and $v \in V_2$ or $x, y \in V_2$ and $v \in V_1$. The aim of paper is to give some characteristic properties of bipartite threshold graphs. In particular, every such graph (V_1, E, V_2) is embedded in the threshold graph $(K(V_1), E, V_2)$, where $K(V_1)$ is the complete graph on the vertex set V_1. Note that a graph is a threshold graph if and only if it has no lifting triples of vertices. Every bipartite graph can be obtained from a bipartite threshold graph by means of lowering edge rotations. Using the obtained results and Kohnert's criterion for a partition to be graphical, we give a new simple proof of the well-known Gale–Ryser theorem on the representation of two partitions by degree partitions of the parts in a bipartite graph.

Keywords: integer partition, threshold graph, bipartite graph, Ferrer's diagram.

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