ESTIMATES FOR THE BEST APPROXIMATIONS OF FUNCTIONS FROM THE NIKOL'SKII–BESOV CLASS IN THE LORENTZ SPACE BY TRIGONOMETRIC POLYNOMIALS

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We consider spaces of periodic functions of many variables, specifically, the Lorentz space $L_{p,\tau}(\mathbb{T}^m)$ and the Nikol'skii–Besov space $S^{\bar{r}}_{p,\tau,\theta}B$, and study the best approximation of a function $f \in L_{p,\tau}(\mathbb{T}^m)$ by trigonometric polynomials with the numbers of harmonics from a step hyperbolic cross. Sufficient conditions are established for a function $f \in L_{p,\tau_1}(\mathbb{T}^m)$ to belong to a space $L_{q,\tau_2}(\mathbb{T}^m)$ in the cases $1 < p < q < \infty$, $1 < \tau_1, \tau_2 < \infty$ and $p = q$, $1 < \tau_2 < \tau_1 < \infty$. Estimates for the best approximations of functions from the Nikol'skii–Besov class $S^{\bar{r}}_{p,\tau_1,\theta}B$ in the norm of the space $L_{q,\tau_2}(\mathbb{T}^m)$ are derived for different relations between the parameters p, q, τ_1, τ_2, and θ. For some relations between these parameters, it is shown that the estimates are exact.

Keywords: Lorentz space, Nikol'skii–Besov class, trigonometric polynomial, best approximation, hyperbolic cross.

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