

MSC: 41A63, 41A99, 26C05

DOI: 10.21538/0134-4889-2020-26-2-47-55

BEST L^2 -EXTENSION OF ALGEBRAIC POLYNOMIALS FROM THE UNIT EUCLIDEAN SPHERE TO A CONCENTRIC SPHERE

V. V. Arestov, A. A. Seleznev

We consider the problem of extending algebraic polynomials from the unit sphere of a Euclidean space of dimension $m \geq 2$ to a concentric sphere of radius $r \neq 1$ with the smallest value of the L^2 -norm. An extension of an arbitrary polynomial is found. As a result, we obtain the best extension of a class of polynomials of given degree $n \geq 1$ whose norms in the space L^2 on the unit sphere do not exceed 1. We show that the best extension equals r^n for $r > 1$ and r^{n-1} for $0 < r < 1$. We describe the best extension method. A.V. Parfenenkov obtained in 2009 a similar result in the uniform norm on the plane ($m = 2$).

Keywords: polynomial, Euclidean sphere, L^2 -norm, best extension.

REFERENCES

1. Parfenenkov A.V. The best extension of algebraic polynomials from the unit circle. *Proc. Steklov Inst. Math.*, 2009, vol. 265, suppl. 1, pp. 194–204. doi: 10.1134/S0081543809060157.
2. Fikhtengol'ts G.M. *Kurs differentsial'nogo i integral'nogo ischisleniya* [A course in differential and integral calculus], vol. 3. Moscow: Fizmatlit Publ., 2002, 728 p. ISBN: 5-9221-0158-7.
3. Deikalova M.V. The Taikov functional in the space of algebraic polynomials on the multidimensional Euclidean sphere. *Math. Notes*, 2008, vol. 84, pp. 498–514. doi: 10.1134/S0001434608090228.
4. Sobolev S.L. *Cubature formulas and modern analysis: An introduction*. Montreux: Gordon and Breach, 1992, 379 p. ISBN: 9782881248412. Original Russian text published in Sobolev S.L. *Vvedenie v teoriyu kubaturnykh formul*. Moscow: Nauka Publ., 1974, 808 p.
5. Stein E.M., Weiss G. *Introduction to Fourier Analysis on Euclidean Spaces*. Princeton: Princeton Univ. Press, 1971, 312 p. ISBN: 9781400883899. Translated to Russian under the title *Vvedenie v garmonicheskii analiz na evklidovykh prostranstvakh*. Moscow: Mir Publ., 1974, 338 p.

Received January 10, 2020; Revised February 10, 2020; Accepted February 17, 2020

Funding Agency: This work was supported by the Russian Academic Excellence Project (agreement no. 02.A03.21.0006 of August 27, 2013, between the Ministry of Education and Science of the Russian Federation and Ural Federal University). The research of the first author was also supported by the Russian Foundation for Basic Research (project no. 18-01-00336).

Vitalii Vladimirovich Arestov, Dr. Phys.-Math. Sci., Ural Federal University, Yekaterinburg, 620083 Russia; Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: vitalii.arestov@urfu.ru.

Anton Aleksandrovich Seleznev, Ural Federal University, Yekaterinburg, 620083 Russia, e-mail: misterion3000@gmail.com.

Cite this article as: V. V. Arestov, A. A. Seleznev. Best L^2 -extension of algebraic polynomials from the unit Euclidean sphere to a concentric sphere. *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2020, vol. 26, no. 2, pp. 47–55.