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**ON THE CONSTRUCTION OF AN INTEGRABLE SOLUTION
TO ONE CLASS OF NONLINEAR INTEGRAL EQUATIONS
OF HAMMERSTEIN–NEMYTSKII TYPE
ON THE WHOLE AXIS**

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We study one class of nonlinear integral equations of convolution type with the Hammerstein–Nemytskii operator on the whole axis. This class has direct applications in the kinetic theory of gases, the theory of p -adic open-closed strings, and the theory of radiative transfer. We prove a constructive theorem on the existence of a nontrivial nonnegative solution integrable on the whole axis. In the end of the paper, we give specific examples of such equations satisfying all conditions of the main theorem.

Keywords: Hammerstein–Nemytskii equations, successive approximations, monotonicity, convexity, convergence of iterations.

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