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MEAN-SQUARE APPROXIMATION OF FUNCTIONS ON THE WHOLE AXIS BY ALGEBRAIC POLYNOMIALS WITH THE CHEBYSHEV–HERMITE WEIGHT

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We derive exact inequalities of Jackson–Stechkin type between the value $E_{n-1}(f^{(s)})_2$ of the best mean-square approximation on \mathbb{R} with the weight $\rho(x) = e^{-x^2}$ of successive derivatives $f^{(s)}$, $s = 0, 1, \dots, r$, of functions $f \in L_{2,\rho}^{(r)}(\mathbb{R})$ and average values of m th-order generalized moduli of continuity of the r th derivatives. The exact values of some extremal approximation characteristics in the space $L_{2,\rho}(\mathbb{R})$ are found for classes of functions defined in terms of these moduli of continuity.

Keywords: best approximations, algebraic polynomial, Jackson–Stechkin inequalities, m th-order modulus of continuity, Chebyshev–Hermite polynomial.

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