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ON THE REGULARIZATION OF THE CLASSICAL OPTIMALITY CONDITIONS IN CONVEX OPTIMAL CONTROL PROBLEMS

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We consider a regularization of the classical optimality conditions (COCs) in a convex optimal control problem for a linear system of ordinary differential equations with a pointwise state equality constraint and a finite number of functional constraints in the form of equalities and inequalities. The set of admissible controls of the problem is traditionally embedded in the space of square integrable functions. However, the objective functional is not, generally speaking, strongly convex. The proof of regularized COCs is based on the use of two regularization parameters. One of them is "responsible" for the regularization of the dual problem, while the other is contained in a strongly convex regularizing addition to the objective functional of the original problem. The main purpose of the regularized Lagrange principle and Pontryagin maximum principle is the stable generation of minimizing approximate solutions in the sense of J. Warga. The regularized COCs: (1) are formulated as theorems on the existence of minimizing approximate solutions in the original problem with the simultaneous constructive presentation of their specific representatives; (2) are expressed in terms of regular classical Lagrange and Hamilton–Pontryagin functions; (3) are sequential generalizations of their classical counterparts and retain their general structure; (4) "overcome" the properties of ill-posedness of COCs and are regularizing algorithms for optimization problems.

Keywords: convex optimal control, convex programming, minimizing sequence, regularizing algorithm, Lagrange principle, Pontryagin maximum principle, dual regularization.

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