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ON THE GIBBS PHENOMENON FOR RATIONAL SPLINE FUNCTIONS

A.-R. K. Ramazanov, A. K. Ramazanov, V. G. Magomedova

In the case of functions $f(x)$ continuous on a given closed interval $[a,b]$ except for jump discontinuity points, the Gibbs phenomenon is studied for rational spline functions $R_{N,1}(x) = R_{N,1}(x,f,\Delta,g)$ defined for a knot grid $\Delta : a = x_0 < x_1 < \cdots < x_N = b$ and a family of poles $g_i \notin [x_{i-1}, x_{i+1}]$ $(i = 1,2,\ldots,N-1)$ by the equalities $R_{N,1}(x) = [R_i(x)(x - x_{i-1}) + R_{i-1}(x)(x_i - x)]/(x_i - x_{i-1})$ for $x \in [x_{i-1}, x_i]$ $(i = 1,2,\ldots,N)$. Here the rational functions $R_i(x) = \alpha_i + \beta_i(x - x_i) + \gamma_i/(x - g_i)$ $(i = 1,2,\ldots,N-1)$ are uniquely defined by the conditions $R_i(x_j) = f(x_j)$ $(j = i-1, i, i+1)$; we assume that $R_0(x) \equiv R_1(x)$, $R_N(x) \equiv R_{N-1}(x)$. Conditions on the knot grid Δ are found under which the Gibbs phenomenon occurs or does not occur in a neighborhood of a discontinuity point.

Keywords: interpolation spline, rational spline, Gibbs phenomenon.

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A.-R.K. Ramazanov, Dr.Phys.-Math., Prof., Dagestan State University, the Republic of Dagestan, Makhachkala, 367002 Russia; Dagestan Scientific Center RAN, the Republic of Dagestan, Makhachkala, 367025 Russia, e-mail: ar-ramazanov@rambler.ru .

A.K. Ramazanov, Cand. Sci. (Phys.-Math.), Bauman Moscow State Technical University (Kaluga Branch), Kaluga, 248000, Russia, e-mail: akramazanov@mail.ru .

V.G. Magomedova, Cand. Sci. (Phys.-Math.), Dagestan State University, the Republic of Dagestan, Makhachkala, 367002 Russia, e-mail: vazipat@rambler.ru .

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