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ON THE CONNECTION BETWEEN THE SECOND DIVIDED DIFFERENCE AND THE SECOND DERIVATIVE

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We formulate the general problem of the extremal interpolation of real-valued functions with the nth derivative defined almost everywhere on the axis $\mathbb{R}$ (for finite differences, this is the Yanenko–Stechkin–Subbotin problem). It is required to find the smallest value of this derivative in the uniform norm on the class of functions interpolating any given sequence $y = \{y_k\}_{k=-\infty}^{\infty}$ of real numbers on an arbitrary, infinite in both directions node grid $\Delta = \{x_k\}_{k=-\infty}^{\infty}$ for a class of sequences Y such that the moduli of their nth-order divided differences on this node grid are upper bounded by a fixed positive number. We solve this problem in the case $n = 2$. For the value of the second derivative according to Yu. N. Subbotin's scheme, we derive upper and lower estimates, which coincide for a geometric node grid of the form $\Delta_p = \{p^k h\}_{k=-\infty}^{\infty}$ ($h > 0$, $p \geq 1$). The estimates are derived in terms of the ratios of neighboring steps of the gird and interpolated values.

Keywords: interpolation, divided difference, splines, derivatives.

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