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ON THE EQUIVALENCE OF REPRODUCING KERNEL HILBERT SPACES CONNECTED BY A SPECIAL TRANSFORM

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We consider two reproducing kernel Hilbert spaces H_1 and H_2 consisting of complex-valued functions defined on some sets of points $\Omega_1 \subset \mathbb{C}^n$ and $\Omega_2 \subset \mathbb{C}^m$, respectively. The norms in the spaces H_1 and H_2 have an integral form:

$$\|f\|_{H_1}^2 = \int_{\Omega_1} |f(t)|^2 d\mu_1(t), \quad f \in H_1, \quad \|q\|_{H_2}^2 = \int_{\Omega_2} |q(z)|^2 d\mu_2(z), \quad q \in H_2.$$

Let $\{E(\cdot, z)\}_{z \in \Omega_2}$ be some complete system of functions in the space H_1 . Define

$$\begin{aligned} \tilde{f}(z) &\stackrel{\text{def}}{=} (E(\cdot, z), f)_{H_1} \quad \forall z \in \Omega_2, \quad \tilde{H}_1 = \{\tilde{f}, f \in H_1\}, \\ (\tilde{f}_1, \tilde{f}_2)_{\tilde{H}_1} &\stackrel{\text{def}}{=} (f_2, f_1)_{H_1}, \quad \|\tilde{f}_1\|_{\tilde{H}_1} = \|f_1\|_{H_1} \quad \forall \tilde{f}_1, \tilde{f}_2 \in \tilde{H}_1. \end{aligned}$$

We prove that the Hilbert spaces \tilde{H}_1 and H_2 are equivalent (i.e., consist of the same functions and have equivalent norms) if and only if there exists a linear continuous one-to-one operator \mathcal{A} acting from the space \tilde{H}_1 onto the space H_2 that for any $\xi \in \Omega_1$ takes the function $K_{\tilde{H}_1}(\cdot, \xi)$ to the function $E(\xi, \cdot)$, where \overline{H}_1 is the space consisting of functions that are complex conjugate to functions from H_1 and $K_{\overline{H}_1}(t, \xi)$, $t, \xi \in \Omega_1$, is the reproducing kernel of \overline{H}_1 . We also obtain other conditions for the equivalence of the spaces \tilde{H}_1 and H_2 . In addition, we study the question of the equivalence of the spaces \tilde{H}_2 and H_1 and the question of the existence of special orthosimilar expansion systems in the spaces H_1 and H_2 . We derive a necessary and sufficient condition for the equivalence of the spaces H_1 and H_2 . This paper continues the authors' paper in which the case of coinciding spaces \tilde{H}_1 and H_2 was considered.

Keywords: orthosimilar decomposition systems, reproducing kernel Hilbert space, problem of describing the dual space.

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