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SOLUTION OF BOUNDARY VALUE PROBLEMS WITH MOVING BOUNDARIES BY AN APPROXIMATE METHOD FOR CONSTRUCTING SOLUTIONS OF INTEGRO-DIFFERENTIAL EQUATIONS

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The problem of oscillations of objects with moving boundaries formulated as a differential equation with boundary and initial conditions is a nonclassical generalization of the hyperbolic type problem. To facilitate the construction of a solution to this problem and justify the choice of the solution form, we construct equivalent integro-differential equations with symmetric time-dependent kernels and time-varying integration limits. The advantages of the method of integro-differential equations are found in the transition to more complex dynamic systems that carry concentrated masses oscillating under mobile loads. The method is extended to a broader class of model boundary value problems that take into account the bending stiffness, environmental resistance, and stiffness of the base of the oscillating object. Special attention is paid to the analysis of the most common applied case when the boundaries are subject to external perturbations. The problem is solved in dimensionless variables up to the values of the second order of smallness relative to the small parameters that characterize the speed of the boundary movement. We find an approximate solution of a problem on transverse vibrations of a rope with bending stiffness in a lifting device; one end of the rope is wound on a drum and the other is fixed to a load. The results obtained for the oscillation amplitude corresponding to the nth dynamic mode are presented. The phenomena of steady-state resonance and passage through the resonance are studied by numerical methods.

Keywords: resonance properties, oscillations in systems with moving boundaries, laws of motion of the boundaries, integro-differential equations, amplitude of oscillations.

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