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SOME PROPERTIES OF POWER OPERATOR SERIES

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A linear densely defined operator and a domain lying in its regular set and containing the nonpositive real semiaxis are given in a Banach space. A power bound for the norm of the resolvent of the operator at infinity is assumed to be known. We consider the question of (left, right) multiplication of a function of an operator, in particular, a complex degree of an operator, by a power operator series and the connection between the domain of this product and the domain of the power operator series. The case of the continuity of the operator function or its inverse and the possibility of taking the function under the series sign are considered separately. In some of the statements proved, certain constraints are imposed on the coefficients of the power series. Examples connected with these constraints and the constraints on the scalar function generating the operator function are analyzed.

Keywords: linear closed operator, functions of an operator, power operator series.

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