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FINITE GROUPS WHOSE MAXIMAL SUBGROUPS ARE SOLVABLE OR HAVE PRIME POWER INDICES

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It is well known that all maximal subgroups of a finite solvable group are solvable and have prime power indices. However, the converse statement does not hold. Finite nonsolvable groups in which all local subgroups are solvable were studied by J. Thompson (1968). R. Guralnick (1983) described all the pairs (G, H) such that G is a finite nonabelian simple group and H is a subgroup of prime power index in G. Several authors studied finite groups in which every subgroup of non-prime-power index (not necessarily maximal) is a group close to nilpotent. Weakening the conditions, E. N. Bazhanova (Demina) and N. V. Maslova (2014) considered the class $\mathfrak{J}_{\rm pr}$ of finite groups in which all nonsolvable maximal subgroups have prime power indices and, in particular, described possibilities for nonabelian composition factors of a nonsolvable group from the class $\mathfrak{J}_{\rm pr}$. In the present note, the authors continue the study of the normal structure of a nonsolvable group from $\mathfrak{J}_{\rm pr}$. It is proved that a group from $\mathfrak{J}_{\rm pr}$ contains at most one nonabelian chief factor and, for each positive integer n, there exists a group from $\mathfrak{J}_{\rm pr}$ such that the number of its nonabelian composition factors is at least n. Moreover, all almost simple groups from $\mathfrak{J}_{\rm pr}$ are determined.

Keywords: finite group, maximal subgroup, prime power index, nonsolvable subgroup.

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