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**ON SOME EFFICIENTLY SOLVABLE CLASSES  
OF THE NETWORK FACILITY LOCATION PROBLEM  
WITH CONSTRAINTS ON THE CAPACITIES OF COMMUNICATION LINES**

**E. Kh. Gimadi, O. Yu. Tsidulk**

We study the network facility location problem with constraints on the capacities of communication lines, called Restricted Facility Location Problem (RFLP). It is required to locate facilities at the vertices of a given network graph so as to simultaneously satisfy the demands of the clients, which are also located at the vertices of the network graph, at minimum cost. We consider two statements of the problem: the multiple-allocation RFLP, where the demand of a client can be satisfied jointly by several facilities, and the single-allocation RFLP, where the demand of a client must be entirely satisfied by a single facility. We show that the single-allocation RFLP is NP-hard even if the network is a simple path and strongly NP-hard if the network is a tree. The multiple-allocation RFLP is weakly NP-hard on trees. For this problem we propose a pseudopolynomial algorithm for the case where the network graph has constant treewidth, and show a linear-time algorithm for the case where the network is a simple path.

Keywords: facility location problem, capacities, single-allocation, multiple-allocation, NP-hard problem, tree-width, pseudopolynomial algorithm, polynomial-time algorithm.

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*Edward Khairutdinovich Gimadi*, Dr. Phys.-Math. Sci., Prof., Sobolev Institute of Mathematics; Novosibirsk State University, Novosibirsk, 630990 Russia, e-mail: gimadi@math.nsc.ru.

*Oxana Yurievna Tsidulko*, Cand. Sci. (Phys.-Math.), Sobolev Institute of Mathematics; Novosibirsk State University, Novosibirsk, 630990 Russia, e-mail: tsidulko@math.nsc.ru

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