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OPTIMAL STRATEGIES IN THE TREATMENT OF CANCERS IN THE LOTKA–VOLTERRA MATHEMATICAL MODEL OF COMPETITION

N. L. Grigorenko, E. N. Khailov, E. V. Grigorieva, A. D. Klimenkova

The Lotka–Volterra competition model is applied to describe the interaction between the concentrations of healthy and cancer cell in diseases associated with blood cancer. The model is supplemented with a differential equation characterizing the change in the concentration of a chemotherapeutic drug. The equation contains a scalar bounded control that specifies the intensity of drug intake. We consider the problem of minimizing the weighted difference between the concentrations of cancer and healthy cells at the end time of the treatment period. The properties of an optimal control are established analytically with the use of the Pontryagin maximum principle. We describe situations in which the optimal control is a relay function and situations in which the control may contain a segment with a singular arc in addition to relay segments. The results obtained are confirmed by corresponding numerical calculations.

Keywords: Lotka–Volterra competition model, nonlinear control system, Pontryagin maximum principle, switching function, bang-bang control, singular arc.

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Nikolai Leont'evich Grigorenko, Dr. Phys.-Math. Sci., Prof., Faculty of Computational Mathematics and Cybernetics, Moscow State Lomonosov University, Moscow, 119992, Russia,
e-mail: grigor@cs.msu.su .

Evgenii Nikolaevich Khailov, Cand. Sci. (Phys.-Math.), Faculty of Computational Mathematics and Cybernetics, Moscow State Lomonosov University, Moscow, 119992, Russia,
e-mail: khailov@cs.msu.su .

Ellina Valer'evna Grigorieva, Cand. Sci. (Phys.-Math.), Prof., Department of Mathematics and Computer Sciences, Texas Woman's University, Denton, TX 76204, USA,
e-mail: egrigorieva@mail.twu.edu .

Anna Dmitrievna Klimenkova, undergraduate student, Faculty of Computational Mathematics and Cybernetics, Moscow State Lomonosov University, Moscow, 119992, Russia,
e-mail: klimenkovaad@mail.ru .

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