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CONSTRUCTION OF SOLUTIONS TO CONTROL PROBLEMS FOR FRACTIONAL-ORDER LINEAR SYSTEMS BASED ON APPROXIMATION MODELS

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We consider an optimal control problem for a dynamical system whose motion is described by a linear differential equation with the Caputo fractional derivative of order $\alpha \in (0, 1)$. The time interval of the control process is fixed and finite. The control actions are subject to geometric constraints. The aim of the control is to minimize a given terminal-integral quality index. In order to construct a solution, we develop the following approach. First, from the considered problem, we turn to an auxiliary optimal control problem for a first-order linear system with lumped delays, which approximates the original system. After that, the auxiliary problem is reduced to an optimal control problem for an ordinary differential system. Based on this, we propose a closed-loop scheme of optimal control of the original system that uses the approximating system as a guide. In this scheme, the control in the approximating system is formed with the help of an optimal positional control strategy from the reduced problem. The effectiveness of the developed approach is illustrated by a problem in which the quality index is the norm of the terminal state of the system.

Keywords: optimal control, linear systems, fractional-order derivatives, approximation, time-delay systems, closed-loop control.

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