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ULTRAFILTERS AND MAXIMAL LINKED SYSTEMS

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The structure of ultrafilters of a broadly understood measurable space and of maximal linked systems defined on this space is studied. Bitopological spaces of ultrafilters and maximal linked spaces obtained in both cases by equipping the space with topologies of Wallman and Stone types are considered; the bitopological space of ultrafilters can be considered as a subspace of the bitopological space whose points are maximal linked systems. For an abstract attainability problem with constraints of asymptotic nature, ultrafilters can be used as generalized elements in extension constructions; for the latter case, we present a new implementation that involves the application of linked families of subsets of the set of ordinary solutions in the construction of constraints of asymptotic nature. A natural generalization of the usual “linkedness” is considered, when it is postulated that the intersection of sets of subfamilies of the original family defining the measurable space of cardinality not exceeding a given positive integer is nonempty. For this case, we establish relations connecting ultrafilters and maximal linked systems considered in the specified generalized sense.

Keywords: bitopological space, maximal linked system, topology, ultrafilter.

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