Vol. 26 No. 1 **2020**

MSC: 34A60, 49J52, 49J53

DOI: 10.21538/0134-4889-2020-26-1-256-273

GRADIENT METHOD FOR SOLVING SOME TYPES OF DIFFERENTIAL INCLUSIONS

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We discuss some classes of problems with differential inclusions, for which an efficient algorithm based on the gradient method is developed. The first part of the paper describes an algorithm for solving differential inclusions with a free or a fixed right end and a convex continuous multivalued mapping that admits a support function with a continuous derivative with respect to the phase coordinates. This algorithm reduces the problem under consideration to the problem of minimizing a certain functional in a function space. For this functional, the Gâteaux gradient is obtained and necessary and, in some cases, sufficient minimum conditions are found. Further, the gradient descent method is applied to the functional. In the second part of the paper, the developed approach is illustrated by solving three main classes of differential inclusions: (1) a differential inclusion obtained from a control system with a variable control domain depending on the phase coordinates, (2) a differential inclusion containing the direct sum, union, or intersection of convex sets in the right-hand side, (3) a linear interval system of ODEs considered as a differential inclusion.

Keywords: differential inclusion, Gâteaux gradient, support function, gradient descent method, linear interval system, variable control domain.

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Received Dezember 23, 2019
Revised January 31, 2020
Accepted February 3, 2020

Funding Agency: This work was supported by the Russian Science Found (project no. 18-71-00006).

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Cite this article as: A. V. Fominyh, V. V. Karelin, L. N. Polyakova. Gradient method for solving some types of differential inclusions, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2020, vol. 26, no. 1, pp. 256–273 .