ON TWO-SIDED APPROXIMATIONS OF REACHABLE SETS OF CONTROL SYSTEMS WITH GEOMETRIC CONSTRAINTS ON THE CONTROLS

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We consider a nonlinear control system in Euclidean space on a finite time interval with controls subject to geometric constraints. The question of constructing lower and upper (by inclusion) approximations of reachable sets of this system is studied. Under certain conditions, estimates are obtained for the discrepancy (in the Hausdorff metric) between the lower and upper approximations of the reachable sets.

Keywords: control system, control, differential inclusion, geometric constraints, reachable set, approximation.

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