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ON THE CONSTRUCTION OF A PIECEWISE AFFINE VALUE FUNCTION IN AN INFINITE-HORIZON OPTIMAL CONTROL PROBLEM

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The paper is devoted to the approximate solution of an infinite-horizon optimal control problem for a nonlinear system of differential equations with an integral cost functional. We use the technique of piecewise linearization (“hybridization”) of the original nonlinear system followed by the analysis of the resulting switched system. Then the methods of piecewise affine value and control functions, the method of dynamic programming, and the comparison principle are applied. Two cases are considered sequentially: with continuous piecewise affine value and control functions and with functions admitting discontinuities. In the latter case, it is possible to increase the effectiveness of the proposed approach by allowing gaps. Theorems on sufficient conditions for the solvability of the control problem are formulated and proved. The theorems also provide upper estimates of the minimized functional. Computationally simple algorithms are derived for the construction of estimates of the value function for this problem and of the corresponding feedback control. The operation of the proposed algorithm is demonstrated for a problem of control of a wheeled robot on the plane.

Keywords: nonlinear dynamics, linearization, switched system, optimal control, dynamic programming, piecewise affine value function.

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