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**DIFFERENTIAL INCLUSIONS IN A BANACH SPACE
WITH COMPOSITE RIGHT-HAND SIDE****A. A. Tolstonogov**

A differential inclusion whose right-hand side is the sum of two multivalued mappings is considered in a separable Banach space. The values of one mapping are closed, bounded, not necessarily convex sets. This mapping is measurable in the time variable, is Lipschitz in the state variable, and satisfies the traditional growth condition. The values of the second mapping are closed, convex, not necessarily bounded sets. This mapping is assumed to have a closed graph in the state variable. The remaining assumptions concern the intersection of the second mapping and the multivalued mapping defined by the growth conditions. We suppose that the intersection of the multivalued mappings has a measurable selection and possesses certain compactness properties. An existence theorem is proved for solutions of such inclusions. The proof is based on a theorem proved by the author on continuous selections passing through fixed points of multivalued mappings depending on a parameter with closed, nonconvex, decomposable values and on Ky Fan's famous fixed-point theorem. The obtained results are new.

Keywords: decomposable space, fixed point, continuous selection, weak norm, Aumann integral.

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