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ANALYTIC DESCRIPTION OF A REACHABLE SET FOR THE DUBINS CAR

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A three-dimensional reachable set at an instant is considered for the controlled object “Dubins car” (also known as unicycle). We study the case when turns to both sides are possible. For this case, the authors earlier specified, based on the Pontryagin maximum principle, six types of piecewise constant open-loop controls; one can restrict oneself to these controls when studying the boundary of the reachable set $G(t_f)$ at a given time t_f . These controls were used for the numerical construction of the reachable set and for its visualization. However, no analytic description of the boundary of the set $G(t_f)$ was obtained. In the present paper, we derive formulas for the generally nonconvex two-dimensional sections of the set $G(t_f)$ with respect to the angular coordinate. The structure of the sections is analyzed. For the dynamics in the normalized coordinates, an analytic description is obtained under the assumption that $t_f \leq 2\pi$. For this case, a theorem is formulated on necessary and sufficient conditions for transferring a motion to the boundary of $G(t_f)$. The case $t_f > 2\pi$ is not covered by the analytic description because of the more complex structure of the sections, in particular, due to the loss of connectivity of $G(t_f)$ for some values of t_f .

Keywords: Dubins car, two-sided turns, three-dimensional reachable set, Pontryagin maximum principle, extremal piecewise constant control, sections of the reachable set along the angle coordinate, analytic description of the sections.

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