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CONSTRUCTION OF THE VIABILITY SET IN A PROBLEM OF CHEMOTHERAPY OF A MALIGNANT TUMOR GROWING ACCORDING TO THE GOMPERTZ LAW

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The problem of chemotherapy of a malignant tumor growing according to the Gompertz law is considered. The mathematical model is a system of two ordinary differential equations. We study a problem of optimal control (optimal therapy) aiming at the minimization of the malignant cells in the body at a given terminal time T . The viability set of this problem, i.e., the set of initial states of the model (the volume of the tumor and the amount of the drug in the body) for which an optimal control guarantees that the dynamics of the system up to the time T is compatible with life in terms of the volume of the tumor, is constructed analytically.

Keywords: viability set, optimal control, value function.

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