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ON POLYHEDRAL ESTIMATION OF REACHABLE SETS IN THE "EXTENDED" SPACE FOR DISCRETE-TIME SYSTEMS WITH UNCERTAIN MATRICES AND INTEGRAL CONSTRAINTS

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The problems of reachability and construction of estimates of reachable sets are considered for discrete-time systems with initially linear structure and uncertainties in the initial conditions, matrices, and additive input actions. The uncertainties are restricted by given parallelepiped-valued, interval, and integral nonquadratic constraints, respectively. The systems under consideration turn out to be of bilinear type due to the uncertainty in the matrices. The reachable sets are considered not only in the original space $\mathbb{R}^n$ but also in the "extended" space $\mathbb{R}^{n+1}$, where the last coordinate μ corresponds to the current reserve of the additive input action. An exact description is given for the reachable sets $\mathcal{Z}[k]$ in the "extended" space using multivalued recurrence relations. Here, the representation of sets in the form of the union of their μ-sections is used, and the recurrence relations include operations with sets; one of the operations (multiplication by an interval matrix) acts on each cross-section independently, and another combines the Minkowski sum and the union over cross-sections. The reachable sets $\mathcal{X}[k]$ in $\mathbb{R}^n$ are determined by the cross-sections of $\mathcal{Z}[k]$ corresponding to $\mu = 0$. However, it is usually difficult to calculate $\mathcal{Z}[k]$ exactly from the above relations. Methods are proposed for the construction of parametrized families of external and internal polyhedral estimates of the sets $\mathcal{Z}[k]$ in the form of polytopes of a special type. On this basis, external parallelepiped-valued and internal parallelotope-valued estimates of $\mathcal{X}[k]$ are constructed. All estimates are found by explicit formulas from systems of recurrence relations.

Keywords: reachable set, integral constraints, uncertain matrix, polyhedral estimates, parallelepipeds, parallelotopes.

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