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ESTIMATION OF STATES OF MULTISTAGE STOCHASTIC INCLUSIONS

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Multistage stochastic inclusions of the form $z_k \in H_k(z_{k-1}, \omega)$, where $z_k \in Z_k = X_k Y_k$ and $k \in 1 : N$, are considered. We regard the projection of z_k to X_k as an unobservable state and the projection of z_k to Y_k as an observable state. The element ω belongs to a probability space $(\Omega, \mathcal{F}, P)$, and the multifunction $H_k(z, \cdot)$ is measurable with respect to a σ-algebra $\mathcal{G}_k$. These σ-algebras are supposed to be independent for different k, and their union $\mathcal{F}_k = \sigma\big(\bigcup_{i\in 1:k} \mathcal{G}_i\big) \subset \mathcal{F}$ characterizes an increasing accumulation of information. We consider three ways of estimating the unobservable states based on different methods of forming the set of transition probabilities. It is shown that these ways result in different sets of conditional distributions for the unobservable states of the process. The question of sufficient conditions for the coincidence of the considered filtering schemes is partially studied, and it is proved that, for finite state spaces, these schemes coincide in the case of a nonatomic probability space. A new class of Lebesgue selections is introduced for arbitrary multifunctions and is shown to be nonempty, in particular, for measurable simple rectangles on a nonatomic space. It is proved that the filtering schemes also coincide in the Lebesgue class for simple inclusions and selections defined on a nonatomic probability space.

Keywords: estimation, filtering, stochastic inclusions, selections, transition probabilities, conditional distributions.

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