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ON CHIEF FACTORS OF PARABOLIC MAXIMAL SUBGROUPS OF THE GROUP ${}^2F_4(2^{2N+1})$

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This study continues the author's previous papers where a refined description of the chief factors of a parabolic maximal subgroup contained in its unipotent radical was obtained for all (normal and twisted) finite simple groups of Lie type except for the groups ${}^2F_4(2^{2n+1})$ and $B_l(2^n)$. In present paper, such a description is given the group ${}^2F_4(2^{2n+1})$. We prove a theorem in which, for every parabolic maximal subgroup of ${}^2F_4(2^{2n+1})$, a fragment of the chief series contained in the unipotent radical of this subgroup is given. Generators of the corresponding chief factors are presented in a table.

Keywords: finite simple group, group of Lie type, parabolic maximal subgroup, chief factor, unipotent radical, strong version of the Sims conjecture.

REFERENCES

1. Bourbaki N. *Groupes et algèbres de Lie (Chapt. IV–VI)*. Paris: Hermann, 1968, 282 p. doi: 10.1007/978-3-540-34491-9. Translated to Russian under the title *Gruppy i algebrы Li (glavy IV – VI)*. Moscow: Mir Publ., 1972, 334 p.
2. Vasil'ev A.V. Minimal permutation representations of finite simple exceptional twisted groups. *Algebra and Logic*, 1998, vol. 37, no. 1, pp. 9–20. doi 10.1007/BF02684081.
3. Korableva V.V. Parabolic permutation representations of the groups ${}^2F_4(q)$ and ${}^3D_4(q^3)$. *Math. Notes*, 2000, vol. 67, no. 1, pp. 55–60. doi: 10.1007/BF02675792.
4. Korableva V.V. On the chief factors of parabolic maximal subgroups of finite simple groups of normal Lie type. *Sib. Math. J.*, 2014, vol. 55, no. 4, pp. 622–638. doi: 10.1134/S0037446614040053.
5. Korableva V.V. On chief factors of parabolic maximal subgroups of the group ${}^2E_6(q^2)$. *International Conf. Algebra and Mathematical Logic: Theory and Applications*, Collection of Abstracts, Kazan: 2014, pp. 82–83 (in Russian).
6. Korableva V.V. On chief factors of parabolic maximal subgroups of the group ${}^3D_4(q^3)$. *International Conf. Mal'tsev Meeting dedicated to 75th anniversary of Yuri L. Ershov*, Collection of Abstracts, Novosibirsk: 2015. P. 106 (in Russian).
7. Korableva V.V. On the chief factors of maximal parabolic subgroups of twisted classical groups. *Sib. Math. J.*, 2015, vol. 56, no. 5, pp. 879–887. doi 10.1134/S0037446615050109.
8. Korableva V.V. On the chief factors of parabolic maximal subgroups of special finite simple groups of exceptional Lie type. *Sib. Math. J.*, 2017, vol. 58, no. 6, pp. 1034–1041. doi 10.1134/S003744661706012X.
9. Azad H., Barry M., Seitz G. *On the structure of parabolic subgroup*. *Comm. Algebra*. 1990. vol. 18, no. 2. pp. 551–562. doi 10.1080/00927879008823931.
10. Carter R.W. *Simple groups of Lie type*. London: John Wiley and Sons, 1972. 331 p.
11. Conway J.H., Curtis R.T., Norton S.P., Parker R.A., Wilson R.A. *Atlas of finite groups*. Oxford: Clarendon Press, 1985, 252 p. ISBN: 0198531990.
12. Fong P., Seitz G. Groups with (B,N)-pair of rank 2, II. *Invent.Math.* 1974. vol. 24. pp. 191–239.
13. Kondrat'ev A.S., Trofimov V.I. Vertex stabilizers of vertices of graphs with primitive automorphism groups and a strong version of the Sims conjecture. *Proc. conf. "Groups St Andrews 2017 in Birmingham"*. Cambridge: Cambridge University Press, 2019 (London Math. Soc. Note Ser. 2019. Vol. 455. pp. 419–426.)
14. Malle G. The Maximal subgroups of ${}^2F_4(q^2)$. *J. Algebra*, 1991, vol. 139, pp. 52–69. doi 10.1016/0021-8693(91)90283-E.

15. Ree R. A family of simple groups associated with simple Lie algebra type F_4 . *Amer. J. Math.*, 1961, vol. 83, pp. 401–420. doi 10.2307/2372886 .
16. Shinoda K. A characterization of odd order extensions of the Ree groups ${}^2F_4(q)$. *J. Fac. Sci. Univ*, 1975, vol. 22, pp. 79–102.
17. Tits J. Algebraic and abstract simple groups. *Ann. of Math.*, 1964, vol. 80, pp. 313–329. doi 10.2307/1970394 .

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