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ON PRIMITIVE PERMUTATION GROUPS WITH THE STABILIZER OF TWO POINTS NORMAL IN THE STABILIZER OF ONE OF THEM: THE CASE WHEN THE SOCLE IS A POWER OF A GROUP $E_8(Q)$

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Assume that G is a primitive permutation group on a finite set X, $x \in X \setminus \{x\}$, and $G_{x,y} \trianglelefteq G_x$. P. Cameron raised the question about the validity of the equality $G_{x,y} = 1$ in this case. The author proved earlier that, if the socle of G is not a power of a group isomorphic to $E_8(q)$ for a prime power q, then $G_{x,y} = 1$. In the present paper, we consider the case where the socle of G is a power of a group isomorphic to $E_8(q)$. Together with the previous result, we establish the following two statements. 1. Let G be an almost simple primitive permutation group on a finite set X. Assume that, if the socle of G is isomorphic to $E_8(q)$, then G_x for $x \in X$ is not the Borovik subgroup of G. Then the answer to Cameron's question for such primitive permutation groups is affirmative. 2. Let G be a primitive permutation group on a finite set X with the property $G \leq H \mathrm{wr} S_m$. Assume that, if the socle of H is isomorphic to $E_8(q)$, then the stabilizer of a point in the group H is not the Borovik subgroup of H. Then the answer to Cameron's question for such primitive permutation groups is also affirmative.

Keywords: primitive permutation group, regular suborbit.

REFERENCES

1. Benson D., Carlson J. Nilpotent elements in the Green ring. *J. Algebra*, 1986, vol. 104, no. 2, pp. 329–350. doi: 10.1016/0021-8693(86)90219-X .
2. Borovik A. V. A maximal subgroup in the simple finite group $E_8(q)$ *Contemporary Mathematics (131)*, 1992, part 1, pp. 67–79. doi: 10.1090/conm/131.1 .
3. Cameron P.J. Suborbits in transitive permutation groups. In: *Combinatorics: Proc. NATO Advanced Study Inst.* (Breukelen, 1974), Part 3: Combinatorial Group Theory, Math. Centre Tracts, vol. 57. Amsterdam: Math. Centrum, 1974, pp. 98–129.
4. Cohen A.M., Liebeck M.W., Saxl J., Seitz G.M. The local maximal subgroups of exceptional groups of Lie type, finite and algebraic. *Proc. London Mat. Soc. (3)*, 1992, vol. 64, no. 1, pp. 21–48. doi: 10.1112/plms/s3-64.1.21 .
5. Conway J.H. et. al. *Atlas of finite groups*. Oxford: Clarendon Press, 1985, 252 p. ISBN: 0-19-853199-0 .
6. Craven D.A. On tensor products of simple modules for simple groups. *Algebras and Representation Theory*, 2013, vol. 16, iss. 2, pp. 377–404. doi: 10.1007/s10468-011-9311-5 .
7. Doty S., Henke A. Decomposition of tensor products of modular irreducibles for SL_2. *Q.J. Math. Algebra*, 2005, vol. 56, no. 2, pp. 189–207. doi: 10.1093/qmath/hah027 .
8. Feit W. *The representation theory of finite groups*. North Holland: Elsevier, 1982, 501 p. ISBN: 978-0-444-86155-9 .
9. Fomin A.N. Properties of suborbits of finite primitive permutation groups. In: Starostin A.I. (ed.), *Group-theoretic studies*, Collection of scientific works, Sverdlovsk: Ural'skoe Otdelenie AN SSSR Publ., 1990, pp. 87–94.
10. Humphreys J.E. *Modular representations of finite groups of Lie type*. Cambridge: Cambridge University Press, 2011, 206 p. doi: 10.1017/CBO9780511525940 .
11. Konygin A.V. On primitive permutation groups with a stabilizer of two points normal in the stabilizer of one of them. *Sib. Elektron. Mat. Izv.*, 2008, vol. 5, pp. 387–406 (in Russian).
12. Konygin A.V. On Cameron's question about primitive permutation groups with stabilizer of two points that is normal in the stabilizer of one of them. *Proc. Steklov Inst. Math. (Suppl.)*, 2014, vol. 285, pp. 116–127.

13. Knapp W. Primitive Permutationsgruppen mit einem Subkonstituenten, dessen Stabilisatorgruppe Fittingfrei ist. *Arch. Math.*, 1974, vol. 25, no. 1, pp. 472–475. doi: 10.1007/BF01238709 .
14. Knapp W. Some problems of Wielandt revisited. *J. Algebra*, 2006, vol. 302, no. 1, pp. 167–185. doi: 10.1016/j.jalgebra.2005.09.013 .
15. Liebeck M.W., Praeger Ch.E., Saxl J. On the O'Nan–Scott theorem for finite primitive permutation groups. *J. Austral. Math. Soc. Ser. A*, 1988, vol. 44, no. 3, pp. 389–396. doi: 10.1017/S144678870003216X .
16. Liebeck M.W., Saxl J., Seitz G.M. Subgroups of maximal rank in finite exceptional groups of Lie type. *Proc. London Math. Soc. (3)*, 1992, vol. 65, no. 2, pp. 297–325. doi: 10.1112/plms/s3-65.2.297 .
17. Liebeck M.W., Seitz G.M. Maximal subgroups of exceptional groups of Lie type, finite and algebraic. *Geom. Dedicata*, 1990, vol. 35, no. 1-3, pp. 353–387. doi: 10.1007/BF00147353 .
18. Liebeck M.W., Seitz G.M. *The maximal subgroups of positive dimension in exceptional algebraic groups.* Mem. Amer. Math. Soc., vol. 169, no. 802. Providence: AMS, 2004, 227 p. ISBN: 0-8218-3482-7 .
19. Lubeck F. Small degree representations of finite Chevalley groups in defining characteristic. *LMS J. Comput. Math.*, 2001, vol. 4, pp. 135–169. doi: 10.1112/S1461157000000838 .
20. *Malle G., Testerman D.* Linear algebraic groups and finite groups of Lie type. Cambridge: Cambridge University Press, 2011. 309 p. doi: 10.1017/CBO9780511994777 .
21. Rietz H.L. On primitive groups of odd order. *Amer. J. Math.*, 1904, vol. 26, no. 1, pp. 1–30. doi: 10.2307/2369903 .
22. Seitz G.M. *Maximal subgroups of exceptional algebraic groups.* Mem. Amer. Math. Soc., vol. 90, no. 441. Providence: AMS, 1991, 197 p. ISBN: 0821825046 .
23. *Unsolved problems in group theory. The Kourovka notebook*, Khukhro E.I., Mazurov V.D. (eds), no. 19. Novosibirsk: Russian Academy of Sciences, Siberain Branch, Sobolev Institute of Mathematics, 2018, 248 p. Available at: ArXiv:1401.0300v13 [math.GR] June 2018.
24. Weiss M.J. On simply transitive groups. *Bull. Amer. Math. Soc.*, 1934, vol. 40, no. 6, pp. 401–405. doi: 10.1090/S0002-9904-1934-05871-3 .
25. Wielandt H. *Finite permutation groups.* N Y: Acad. Press, 1964, 114 p. ISBN: 9781483258294 .

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