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ON SOME GROUPS OF 2-RANK 1

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The structure of finite groups of 2-rank 1 is largely defined by the classical Burnside and Brauer–Suzuki theorems. Burnside proved that all elements of odd order of a finite group with a cyclic 2-Sylow subgroup form a normal subgroup. S.I. Adyan showed that this statement does not hold in the class of periodic groups even in the case when a Sylow 2-subgroup has order 2 and coincides with the center of the group. The results of Burnside, Brauer, and Suzuki can be formulated as one theorem: in a finite group G of 2-rank 1, the image of any involution in the quotient group $G/O(G)$ lies in the center of this quotient group. It is unknown whether the same statement holds for a periodic group G (V.P. Shunkov's Question 4.75 from the "Kourovka Notebook"). There is no answer even when the centralizer of the involution i is a locally cyclic group (V.D. Mazurov's Question 15.54 from the "Kourovka Notebook"). In Theorem 1, we give a partial affirmative answer to Question 4.75 under an additional condition: in the group G an involution i generates a finite subgroup with any element of order not divisible by 4. In particular, Question 4.75 is solved positively in the classes of binary finite and conjugate binary finite groups. In Theorem 2, we study the structure of a nonlocally finite group G with a finite involution and an involution i whose centralizer is a locally cyclic 2-group. An involution i of a group G is called *finite* if the subgroup $\langle i, i^g\rangle$ is finite for every $g \in G$. In particular, Theorem 2 defines the structure of a counterexample (under the assumption of its existence) to Question 15.54.

Keywords: group of 2-rank 1, periodic group, locally finite group, finite involution.

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