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**THEOREMS ON PERTURBATIONS OF COVERING MAPPINGS IN SPACES
WITH A DISTANCE AND IN SPACES WITH A BINARY RELATION****S. Benarab, E. S. Zhukovskii, W. Merchela**

Statements on the existence of solutions of special-type equations in spaces with a distance and in spaces with a binary relation are derived. The results obtained generalize the well-known theorems on coincidence points of a covering and a Lipschitz mappings and on Lipschitz perturbations of covering mappings in metric spaces as well as the theorems on coincidence points of a covering and an isotonic mappings and on antitone perturbations of covering mappings in partially ordered spaces. In the first part of the paper, we consider a mapping $F: X \times X \rightarrow Y$, where X is a metric space and Y is equipped with a distance satisfying only the identity axiom. “Weakened analogs” of the notions of covering and Lipschitz mappings from X to Y are defined. Under the assumption that F is covering in the first argument and Lipschitz in the second argument (in the sense of the definitions of these properties given in the paper), the existence of a solution x to the equation $F(x, x) = y$ is established. It is shown that this statement yields conditions for the existence of a coincidence point of a covering and a Lipschitz mappings acting from X to Y . In the second part of the paper, similar results are obtained in the case when X is a partially ordered space and Y is equipped with a reflexive binary relation (which is neither transitive nor antisymmetric). “Weakened analogs” of the notions of ordered covering and monotonicity of mappings from X to Y are defined. Under the assumption that F is covering in the first argument and antitone in the second argument (in the sense of the definitions of these properties given in the paper), the existence of a solution x to the equation $F(x, x) = y$ is established and conditions for the existence of a coincidence point of a covering and an isotone mappings acting from X to Y are deduced from this statement. In the third part, a connection between the obtained statements is established. Namely, it is proved that the theorem on the solvability of an operator equation in spaces with a binary relation implies a similar theorem in spaces with a distance and, accordingly, the statements on coincidence points.

Keywords: metric space, ordered space, covering mapping, Lipschitz mapping, monotone mapping.

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