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ON THE CONJUGACY OF THE SPACE OF MULTIPLIERS

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A. Figà Talamanca proved (1965) that the space $M_r = M_r(G)$ of bounded linear operators in the space L_r , $1 \leq r \leq \infty$, on a locally compact group G that are translation invariant (more exactly, invariant under the group operation) is the conjugate space for a space $A_r = A_r(G)$, which he described constructively. In the present paper, for the space $M_r = M_r(\mathbb{R}^m)$ of multipliers of the Lebesgue space $L_r(\mathbb{R}^m)$, $1 \leq r < \infty$, we present a Banach function space $F_r = F_r(\mathbb{R}^m)$ with two properties. The space M_r is conjugate to F_r : $F_r^* = M_r$; actually, it is proved that F_r coincides with $A_r = A_r(\mathbb{R}^m)$. The space F_r is described in different terms as compared to A_r . This space appeared and has been used by the author since 1975 in the studies of Stechkin's problem on the best approximation of differentiation operators by bounded linear operators in the spaces $L_\gamma(\mathbb{R}^m)$, $1 \leq \gamma \leq \infty$.

Keywords: predual space for the space of multipliers.

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