ТРУДЫ ИНСТИТУТА МАТЕМАТИКИ И МЕХАНИКИ УрО РАН

Том 25 № 4 2019

MSC: 05C25

DOI: 10.21538/0134-4889-2019-25-4-44-51

INVERSE PROBLEMS IN THE THEORY OF DISTANCE-REGULAR GRAPHS: DUAL 2-DESIGNS

I. N. Belousov, A. A. Makhnev

Let Γ be a distance-regular graph of diameter 3 with a strongly regular graph Γ_3. Finding the parameters of Γ_3 from the intersection array of Γ is a direct problem, and finding the intersection array of Γ from the parameters of Γ_3 is its inverse. The direct and inverse problems were solved by A. A. Makhnev and M. S. Nirova: if a graph Γ with intersection array $\{k, b_1, b_2; 1, c_2, c_3\}$ has eigenvalue $\theta_2 = -1$, then the graph complementary to Γ_3 is pseudo-geometric for $pG_{c_3}(k, b_1/c_2)$. Conversely, if Γ_3 is a pseudo-geometric graph for $pG_\alpha(k, t)$, then Γ has intersection array $\{k, c_2t, k-\alpha+1; 1, c_2, \alpha\}$, where $k-\alpha+1 \le c_2t < k$ and $1 \le c_2 \le \alpha$. Distance-regular graphs Γ of diameter 3 for which the graph Γ_3 ($\bar{\Gamma}_3$) is pseudogeometric for a net or a generalized quadrangle were studied earlier. In this paper we study intersection arrays of distance-regular graphs Γ of diameter 3 for which the graph Γ_3 ($\bar{\Gamma}_3$) is pseudogeometric for a dual 2-design $pG_{t+1}(l, t)$. New infinite families of feasible intersection arrays are found: $\{m(m^2-1), m^2(m-1), m^2; 1, 1, (m^2-1)(m-1)\}$, $\{m(m+1), (m+2)(m-1), m+2; 1, 1, m^2-1\}$, and $\{2m(m-1), (2m-1)(m-1), 2m-1; 1, 1, 2(m-1)^2\}$, where $m \equiv \pm 1 \pmod 3$. The known families of Steiner 2-designs are unitals, designs corresponding to odd-order projective planes containing a hyperoval, designs of points and lines of projective spaces $PG(n, q)$, and designs of points and lines of affine spaces $AG(n, q)$. We find feasible intersection arrays of a distance-regular graph Γ of diameter 3 for which the graph Γ_3 ($\bar{\Gamma}_3$) is pseudogeometric for one of the known Steiner 2-designs.

Keywords: distance-regular graph, dual 2-design.

REFERENCES

1. Brouwer A.E., Cohen A.M., Neumaier A. *Distance-regular graphs*. Berlin; Heidelberg; N Y: Springer-Verlag, 1989, 495 p. ISBN: 3-540-50619-5 .
2. Koolen J.H., Park J. Shilla distance-regular graphs. *Europ. J. Comb.*, 2010, vol. 31, pp. 2064–2073.
3. Jurisic A., Koolen J. Krein parameters and antipodal tight graphs with diameter 3 and 4. *Discrete Math.*, 2002, vol. 244, pp. 181–202. doi: 10.1016/S0012-365X(01)00082-6 .
4. Bang S., Koolen J. Distance-regular graphs of diameter 3 having eigenvakue -1. *Linear Algebra Appl.*, 2017, vol. 531, pp. 38–53. doi: 10.1016/j.laa.2017.05.038 .
5. Makhnev A.A., Nirova M.S. Distance-regular Shilla graphs with $b_2 = c_2$. *Math. Notes*, 2018, vol. 103, no. 5, pp. 780–792. doi: 10.1134/S0001434618050103 .
6. Barwick S., Ebert G. *Unitals in projective planes*. N Y etc.: Springer, 2008, 193 p. doi: 10.1007/978-0-387-76366-8 .
7. Assmus E.F., Key J.D. Jr. *Designs and their codes*. Chap. 8: Steiner systems. Cambridge: Cambridge Univ. Press, 1994, pp. 295–316. doi: 10.1017/CBO9781316529836.009 .
8. Makhnev A.A., Belousov I.N., Paduchikh D.V. *Konechnye geometrii i ikh avtomorfizmy* [Finite geometries and their automorphisms]. *Novosibirsk: SB RAS Publ.*, 2016, 188. ISBN: 978-5-7692-1521-6 .
9. Bruck R.H., Ryser H.J. The nonexistence of certain finite projective planes. *Canadian J. Math.*, 1949, vol. 1, pp. 88–93. doi: 10.4153/cjm-1949-009-2 .

Received August 1, 2019
Revised November 8, 2019
Accepted November 25, 2019

Ivan Nikolaevich Belousov, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia; Ural Federal University, Yekaterinburg, 620083 Russia, e-mail: i_belousov@mail.ru

Aleksandr Alekseevich Makhnev, Dr. Phys.-Math. Sci., RAS Corresponding Member, Prof., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia; Ural Federal University, Yekaterinburg, 620083 Russia,
e-mail: makhnev@imm.uran.ru

Cite this article as: I. N. Belousov, A. A. Makhnev. Inverse problems in the theory of distance-regular graphs: Dual 2-designs, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2019, vol. 25, no. 4, pp. 44–51 .