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DESCRIPTION OF THE LINEAR PERRON EFFECT UNDER PARAMETRIC PERTURBATIONS EXPONENTIALLY VANISHING AT INFINITY

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Let $\mathcal{M}_n$ be the set of linear differential systems of order $n \geqslant 2$ whose coefficients are continuous and bounded on the time semiaxis $\mathbb{R}_+$. Denote by $\lambda_1(A) \leqslant \ldots \leqslant \lambda_n(A)$ the Lyapunov exponents of a system $A \in \mathcal{M}_n$, by $\Lambda(A) = (\lambda_1(A), \ldots, \lambda_n(A))$ their spectrum, and by $\mathrm{es}(A)$ the exponential stability index of A (the dimension of the linear subspace of solutions with negative characteristic exponents). For a system $A \in \mathcal{M}_n$ and a metric space M, we consider the class $\mathcal{E}_n[A](M)$ of continuous $(n \times n)$ matrix-valued functions $Q\colon \mathbb{R}_+ \times M \to \mathbb{R}^{n\times n}$ satisfying the bound $\|Q(t,\mu)\| \leqslant C_Q \exp(-\sigma_Q t)$ for all $(t,\mu) \in \mathbb{R}_+ \times M$, where C_Q and σ_Q are positive constants (possibly different for each function Q), and such that the Lyapunov exponents of the system $A+Q$, which are functions of $\mu \in M$ and are denoted by $\lambda_1(\mu; A+Q) \leqslant \ldots \leqslant \lambda_n(\mu; A+Q)$, are not less than the corresponding Lyapunov exponents of the system A; i.e., $\lambda_k(\mu; A+Q) \geqslant \lambda_k(A)$, $k = \overline{1,n}$, for all $\mu \in M$. The problem is to obtain a complete description for each $n \in \mathbb{N}$ and each metric space M of the class of pairs $\big(\Lambda(A), \Lambda(\cdot\,; A+Q)\big)$ composed of the spectrum $\Lambda(A) \in \mathbb{R}^n$ of a system $A \in \mathcal{M}_n$ and the spectrum $\Lambda(\cdot\,; A+Q)\colon M \to \mathbb{R}^n$ of a family $A+Q$, where A ranges over $\mathcal{M}_n$ and the matrix-valued function Q ranges over the class $\mathcal{E}_n[A](M)$ for each A, i.e., of the class $\Pi\mathcal{E}_n(M) = \{\big(\Lambda(A), \Lambda(\cdot\,; A+Q)\big) \mid A \in \mathcal{M}_n,\ Q \in \mathcal{E}_n[A](M)\}$. The solution of this problem is provided by the following statement: for each integer $n \geqslant 2$ and every metric space M, a pair $\big(l, F(\cdot)\big)$, where $l = (l_1, \ldots, l_n) \in \mathbb{R}^n$ and $F(\cdot) = (f_1(\cdot), \ldots, f_n(\cdot))\colon M \to \mathbb{R}^n$, belongs to the class $\Pi\mathcal{E}_n(M)$ if and only if the following conditions are met: (1) $l_1 \leqslant \ldots \leqslant l_n$, (2) $f_1(\mu) \leqslant \ldots \leqslant f_n(\mu)$ for all $\mu \in M$, (3) $f_i(\mu) \geqslant l_i$ for all $i = \overline{1,n}$ and $\mu \in M$, (4) for each $i = \overline{1,n}$, the function $f_i(\cdot)\colon M \to \mathbb{R}$ is bounded and, for any $r \in \mathbb{R}$, the preimage $f_i^{-1}([r,+\infty))$ of the half-interval $[r,+\infty)$ is a G_δ-set. The solution of the similar problem of describing the pairs composed of the exponential stability index $\mathrm{es}(A) \in \{0, \ldots, n\}$ of a system A and the exponential stability index $\mathrm{es}(\cdot\,; A+Q)\colon M \to \{0,\ldots,n\}$ of a family $A+Q$, i.e., the class $\mathcal{IE}_n(M) = \{\big(\mathrm{es}(A), \mathrm{es}(\cdot\,; A+Q)\big) \mid A \in \mathcal{M}_n,\ Q \in \mathcal{E}_n[A](M)\}$, is contained in the following statement: for any positive integer $n \geqslant 2$ and every metric space M, a pair $\big(d, f(\cdot)\big)$, where $d \in \{0,\ldots,n\}$ and $f\colon M \to \{0,\ldots,n\}$, belongs to the class $\mathcal{IE}_n(M)$ if and only if $f(\mu) \leqslant d$ for all $\mu \in M$ and, for any $r \in \mathbb{R}$, the preimage $f^{-1}((-\infty, r])$ of the half-interval $(-\infty, r]$ is a G_δ-set.

Keywords: linear differential system, Lyapunov exponents, perturbations vanishing at infinity, Baire classes.

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