TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 25 No. 4 **2019**

MSC: 91B64

DOI: 10.21538/0134-4889-2019-25-4-265-274

MATHEMATICAL MODELING OF INVESTMENTS AT AN IMPERFECT CAPITAL MARKET

A. A. Shananin

We consider the problem of modeling the investments at an imperfect capital market, in which the interest on loans significantly exceeds the interest on deposits. To determine the cash flow deflator, we propose to use the Cantor–Lippman model, in which the investment environment is described by a pool of stationary, replicated projects. The pool of investment projects defines the investment function, which is built as the pointwise maximum of Laplace transforms of the cash flows of investment projects. The Cantor–Lippman model of investment in an imperfect capital market allows us to build a Bellman function, which can be used to assess the financial condition of the investor. We study the properties of the Bellman operator in the problem of an optimal investment strategy. It is shown that the minimum positive root of the investment function should be used as a cash flow deflator. We also study a dynamic control system describing the investment process. Modes of balanced growth are built. The Neumann growth rate and the Neumann equilibrium states are determined. A weak line theorem is proved.

Keywords: investments, Cantor–Lippman model, mathematical modeling of economics, NPV, IRR, Bellman operator, investment polynomial, linear programming.

REFERENCES

1. Fisher I. *The rate of interest*. N Y: Macmillan Co., 1907, 442 p.
2. Fisher I. *The theory of interest*. N Y: Macmillan Co., 1930, 566 p.
3. Hirshleifer J. On the theory of optimal investment decision. *J. Political Economy*, 1958, vol. 66, no. 4, pp. 329–352. doi: 10.1086/258057 .
4. Solow R.M. *Capital theory and the rate of return*. Amsterdam: North Holland Press, 1963, 98 p.
5. Gale D. On the theory of interest. *The American Mathematical Monthly*. 1973, vol. 80, no. 8, pp. 853–868. doi: 10.2307/2319391 .
6. Dorfman R. The meaning of internal rates of return. *J. Finance*, 1981, vol. 36, no. 5, pp. 1011–1021. doi: 10.1111/j.1540-6261.1981.tb01072.x .
7. Cantor D.G., Lipman S.A. Investment selection with imperfect capital markets. *Econometrica*, 1983, vol. 51, no. 4, pp. 1121–1144. doi: 10.2307/1912055 .
8. Cantor D.G., Lipman S.A. Optimal investment selection with a multitude of projects. *Econometrica*, 1995, vol. 63, no. 5, pp. 1231–1240. doi: 10.2307/2171729 .
9. Adler L., Gale D. Arbitrate and growth rate for riskless investments in a stationary economy. *Math. Finance*, 1997, vol. 7, no. 1, pp. 73–81. doi: 10.1111/1467-9965.00023 .
10. Sonin I.M. Growth rate, internal rates of return and turnpikes in an investment model. *Econ. Theory*, 1995, vol. 5, no. 3, pp. 383–400. doi: 10.1007/BF01212325 .
11. Presman E.L., Sonin I.M. *Growth rate, internal rates of return and financial bubbles*. Moscow: TsEMI RAN Publ., 2000, 33 p. ISBN: 5-8211-0122-0 .
12. Belen'kii V.Z. *Ekonomicheskaya dinamika: analiz investitsionnykh proektov v ramkakh lineinoi modeli Neimana–Geila* [Economic dynamics: an analysis of investment projects in the framework of the von Neumann–Gale linear model]. Moscow: TsEMI RAN Publ., 2002, 78 p. ISBN: 5-8211-0212-X .
13. Vashchenko M.P. Investment projects yield estimation under uncertainty. *Math. Models Comput. Simul.*, 2010, vol. 2, no. 1, pp. 33–45. doi: 10.1134/S2070048210010047 .
14. Vashchenko M.P., Shananin A.A. The estimation of the yield of the pool of investment projects in the optimal invetsing problem for continuous time. *Math. Models Comput. Simul.*, 2012, vol. 4, no. 5, pp. 497–508. doi: 10.1134/S2070048212050092 .

15. Shananin A.A., Vashchenko M.P., Zhang Sh. Financial bubbles existence in the Cantor–Lippman model for continuous time. *Lobachevskii J. Math.*, vol. 39, no. 7, pp. 929–935. doi: 10.1134/S1995080218070181 .
16. Rubinov A.M. *Superlineinye mnogoznachnye otobrazheniya i ikh prilozheniya k ekonomicheskim zadacham* [Superlinear many-valued mappings and their application to economical-mathematical problems]. Leningrad: Nauka Publ., 1980, 166 p.

Received October 10, 2019
Revised October 30, 2019
Accepted November 11, 2019

Funding Agency: This work was supported by the Russian Science Foundation (project no. 16-11-10246).

Aleksandr Alekseevich Shananin, Dr. Phys.-Math. Sci, RAS Corresponding Member, Prof., Moscow Institute of Physics and Technology (National Research University), Moscow, 141701 Russia,
e-mail: alexshan@yandex.ru .

Cite this article as: A. A. Shananin. Mathematical modeling of investments at an imperfect capital market, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2019, vol. 25, no. 4, pp. 265–274 .