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SHARP INEQUALITIES OF JACKSON–STECHKIN TYPE FOR PERIODIC FUNCTIONS IN L_2 DIFFERENTIABLE IN THE WEYL SENSE

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For periodic functions differentiable in the sense of Weyl and belonging to the space L_2, sharp inequalities of Jackson–Stechkin type are obtained for a special mth-order modulus of continuity generated by the Steklov operator (function). Similar characteristics of smoothness of functions were considered earlier by V. A. Abilov, F. V. Abilova, V. M. Kokilashvili, S. B. Vakarchuk, V. I. Zabutnaya, K. Tukhliev, etc. For classes of functions defined in terms of these characteristics, we solve a number of extremal problems of polynomial approximation theory.

Keywords: best approximation, periodic function, special modulus of continuity, Jackson–Stechkin inequalities, extremal problems.

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