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## ON PERIODIC GROUPS WITH A REGULAR AUTOMORPHISM OF ORDER 4

A. I. Sozutov

We study periodic groups of the form  $G = F \rtimes \langle a \rangle$  with the conditions  $C_F(a) = 1$  and  $|a| = 4$ . In this case, a finite group  $F$  is solvable and its commutator subgroup is nilpotent (Gorenstein and Herstein, 1961), and a locally finite group  $F$  is solvable and its second commutator subgroup is contained in the center  $Z(F)$  (Kovach, 1961). A locally finite group  $F$  is solvable and its second commutator subgroup is contained in the center  $Z(F)$  (Kovach, 1961). It is unknown whether a periodic group  $F$  is always locally finite (Shumyatskii's Question 12.100 from the Kourovka Notebook). We establish the following properties of groups. For  $\pi = \pi(F) \setminus \pi(C_F(a^2))$ , the group  $F$  is  $\pi$ -closed and the subgroup  $O_\pi(F)$  is abelian and is contained in  $Z([a^2, F])$  (Theorem 1). A group  $F$  without infinite elementary abelian  $a^2$ -admissible subgroups is locally finite (Theorem 2). In a nonlocally finite group  $F$ , there is a nonlocally finite  $a$ -admissible subgroup factorizable by two locally finite  $a$ -admissible subgroups (Theorem 3). For any positive integer  $n$  divisible by an odd prime, we give examples of nonlocally finite periodic groups with a regular automorphism of order  $n$ .

Keywords: periodic group, regular automorphism (fixed-point-free automorphism), solvability, local finiteness, nilpotency.

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*Sozutov Anatolij Ilich*, Dr. Phys.-Math. Sci., Prof., Siberian Federal University, Krasnoyarsk, 660041 Russia, e-mail: sozutov\_ai@mail.ru.

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