ON GENETIC CODES OF CERTAIN GROUPS WITH 3-TRANSPOSITIONS

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Coxeter groups have numerous applications in mathematics and beyond, and B. Fischer's 3-transposition groups underly the internal geometric analysis in the theory of finite (simple) groups. The intersection of these classes of groups consists of finite Weyl groups $W(A_n) \simeq S_{n+1}$, $W(D_n)$, and $W(E_n)$ for $n = 6, 7, 8$, simple finite-dimensional algebras, and Lie groups. In previous papers by A. I. Sozutov, A. A. Kuznetsov, and the author, systems S of generating transvections (3-transpositions) of groups $Sp_{2m}(2)$ and $O_{2m}^{\pm}(2)$ were found such that the graphs $\Gamma(S)$ are trees. A set $\{\Gamma_n\}$, $n \geq m$, of nested graphs is called an *E-series* if these graphs are trees, contain the subgraph E_6, and their subgraphs with vertices $m, m+1, \ldots, n$ are simple chains. In the present paper, we find genetic codes of the groups $Sp_{2m}(2)$ and $O_{2m}^{\pm}(2)$, $8 \leq 2m \leq 20$; these codes are close to the genetic codes of some Coxeter groups. Our main hypothesis is the following: the groups $Sp_{2m}(2)$ and $O_{2m}^{\pm}(2)$ (cases (ii)–(iii) in Fischer's theorem) can be obtained from the corresponding infinite Coxeter groups with the use of one or two additional relations of the form $w^2 = 1$. The graphs I_n considered in this paper contain the subgraph E_6 and comprise an E-series of nested graphs $\{I_n \mid n = 7, 8, \ldots\}$, in which the subgraph $I_n \setminus E_6$ is a simple chain. We prove that the isomorphisms $X(I_{4k+1}) \simeq Sp_{4k}(2) \times Z_2$ and $X(I_{2m}) \simeq O_{2m}^{\pm}(2)$ (the sign $\pm$ depends on m) hold for the groups $X(I_n)$ obtained from the Coxeter groups $G(I_n)$ by imposing an additional relation $(s_4^t s_7)^2 = 1$, where $t = s_3 s_2 s_1 s_5 s_6 s_3 s_2 s_5 s_3 s_4$, if $n = 4k + \delta$ ($\delta = 0, 1, 2$). The proof uses the Todd–Coxeter algorithm from the GAP system.

Keywords:Keywords: genetic code, Coxeter group, Coxeter graph, Weyl group, 3-transposition group, symplectic transvection.

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