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EXTREMAL FUNCTION INTERPOLATION FOR A SECOND-ORDER LINEAR DIFFERENTIAL OPERATOR

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The paper is devoted to the problem of extremal interpolation of functions with the minimum value of the uniform norm of the linear differential operator $\mathcal{L}f(t) = f''(t) + (1/t)f'(t)$ on a class of interpolated values of these functions at the points of a uniform grid $\{kh : k = 1, 2, \ldots, N\}$ with step h $(h > 0)$ for a rather large but finite number N of knots of the grid. The class of interpolation data is defined by a difference analog of the differential operator $\mathcal{L}$. The difference operator is determined by the condition of vanishing of the restrictions of functions from the kernel of the differential operator to the uniform grid. The main result of the paper is a two-sided estimate for an extremal interpolation constant of Subbotin's type with a correct order with respect to the step h. The problem of finding this constant can also be interpreted as a generalized interpolation problem of Favard's type considered on the described class of interpolation data. We use this one-dimensional result to derive an upper bound in a similar problem for the uniform norm of the Laplace operator of a function of two variables in the case of transfinite interpolation at a finite number of concentric circles centered at the origin.

Keywords: interpolation, differential operator, difference operator, Laplace operator.

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