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FINITE GROUPS WITH SUPERSOLUBLE SUBGROUPS OF GIVEN ORDERS

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We study a finite group G with the following property: for any of its maximal subgroups H, there exists a subgroup H_1 such that $|H_1| = |H|$ and $H_1 \in \mathfrak{F}$, where $\mathfrak{F}$ is the formation of all nilpotent groups or all supersoluble groups. We prove that, if $\mathfrak{F} = \mathfrak{N}$ is the formation of all nilpotent groups and G is nonnilpotent, then $|\pi(G)| = 2$ and G has a normal Sylow subgroup. For the formation $\mathfrak{F} = \mathfrak{U}$ of all supersoluble groups and a soluble group G with the above property, we prove that G is supersoluble, or $2 \leq |\pi(G)| \leq 3$; if $|\pi(G)| = 3$, then G has a Sylow tower of supersoluble type; if $|\pi(G)| = 2$, then either G has a normal Sylow subgroup or, for the largest $p \in \pi(G)$, some maximal subgroup of a Sylow p-subgroup is normal in G. If G is nonsoluble and, for each maximal subgroup of G, there exists a supersoluble subgroup of the same order, then every nonabelian composition factor of G is isomorphic to $PSL_2(p)$ for some prime p; we list all such values of p.

Keywords: finite group, soluble group, maximal subgroup, nilpotent subgroup, supersoluble subgroup.

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