

MSC: 41A45, 42B05, 35S05, 58J40

DOI: 10.21538/0134-4889-2019-25-4-15-30

LINEAR RECOVERY OF PSEUDODIFFERENTIAL OPERATORS ON CLASSES OF SMOOTH FUNCTIONS ON AN M -DIMENSIONAL TORUS. II

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We formulate and discuss a problem of optimal recovery of values $T_a f$ of pseudodifferential operators T_a on an m -dimensional torus \mathbb{T}^m with symbols a from the classes $\tilde{\Psi}_{\epsilon, \theta}^{\tau, m}[v; \kappa, L]$ on distributions f from the classes $B_{p, q}^{s, m}(\mathbb{T}^m)$ of Nikol'skii–Besov type and $L_{p, q}^{s, m}(\mathbb{T}^m)$ of Lizorkin–Triebel type from finite spectral information about the symbol of the operator and the distribution (finite sets of Fourier coefficients of the symbol and the distribution). We show that the recovery method $\Upsilon_{A(\gamma, N)}$ constructed and studied in 2018 in the first part of this research is order-optimal (or at least linear order-optimal) in this problem for a number of relations between the parameters of the symbol class, the class of distributions, and the ambient space. Furthermore, the (linear) optimal recovery error has exact order of the corresponding Fourier widths of the classes $B_{p, q}^{s-\tau, m}(\mathbb{T}^m)$ and $L_{p, q}^{s-\tau, m}(\mathbb{T}^m)$, respectively (Theorem 1). Simultaneously, the claim of Theorem 1 from part I of this research is proved under “natural” conditions on the differential parameters τ of the symbol classes $\tilde{\Psi}_{\epsilon, \theta}^{\tau, m}[v; \kappa, L]$ and s of the spaces $B_{p, q}^{s, m}(\mathbb{T}^m)$ of Nikol'skii–Besov type and $L_{p, q}^{s, m}(\mathbb{T}^m)$ of Lizorkin–Triebel type. It is also established that the upper estimates in Theorem 1 are order-exact (see Theorem 3).

Keywords: pseudodifferential operator on an m -dimensional torus, class of symbols (of product type), Nikol'skii–Besov / Lizorkin–Triebel space of distributions, optimal recovery of an operator class, error bounds of optimal recovery, Fourier width.

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Received August 9, 2019

Revised November 8, 2019

Accepted November 25, 2019

Funding Agency: This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan (grant no. AP05133257).

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Cite this article as: D. B. Bazarkhanov. Linear recovery of pseudodifferential operators on classes of smooth functions on an m -dimensional torus. II, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2019, vol. 25, no. 4, pp. 15–30.